

# Phys 23.02 quick lecture

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2025 W34<sup>-1</sup>

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<sup>1</sup>Phys 23.02. All figures are from Young and Freedman (2019) unless noted.

# Chapters [exp #, if applicable]

Units, physical quantities, and vectors [E1] 


Motion along a straight line [E2] 


Motion in two or three dimension [E4] 

Newton's laws of motion 

Applying Newton's laws [E3] 

Work and kinetic energy [E6] 

Potential energy and energy conservation 

Momentum, impulse, and collisions [E5] 

Rotation of rigid bodies ●

Dynamics of rotational motion ●

Equilibrium and elasticity ●

Fluid mechanics [E7] 🌊

Gravitation ●

Periodic motion [E6] 📌

Mechanical waves ●

Sound and hearing ●

Temperature and heat ●

Thermal properties of matter ●

The first law of thermodynamics ●

The second law of thermodynamics ●

Electric charge and electric field ●

Gauss's law ●

Electric potential ●

Capacitance and dielectrics ●

Current, resistance, and electromotive force [E8] ⚡

Direct-current circuits [E8] 💡

Magnetic field and magnetic forces ●

Sources of magnetic field ●

Electromagnetic induction field ●

The nature and propagation of light [E10] 💡

**Units, physical quantities, and vectors [E1]** 

**Motion along a straight line [E2]** 

**Motion in two or three dimension [E4]** 

**Newton's laws of motion ●**

Applying Newton's laws [E3] 🐌

# Work and kinetic energy [E6] 🔗

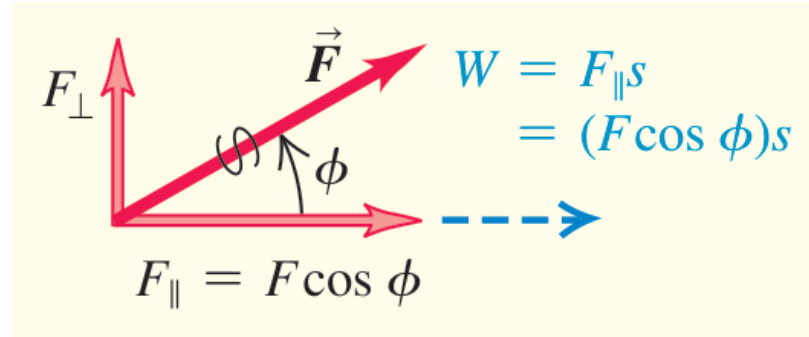


## Work done by a force

- When a constant force  $\vec{F}$  acts on a particle that undergoes a straight-line displacement  $\vec{s}$ , the work done by the force on the particle is defined to be scalar product of  $\vec{F}$  and  $\vec{s}$

$$W = \vec{F} \cdot \vec{s} = F s \cos \phi,$$

$\phi$  = angle between  $\vec{F}$  and  $\vec{s}$



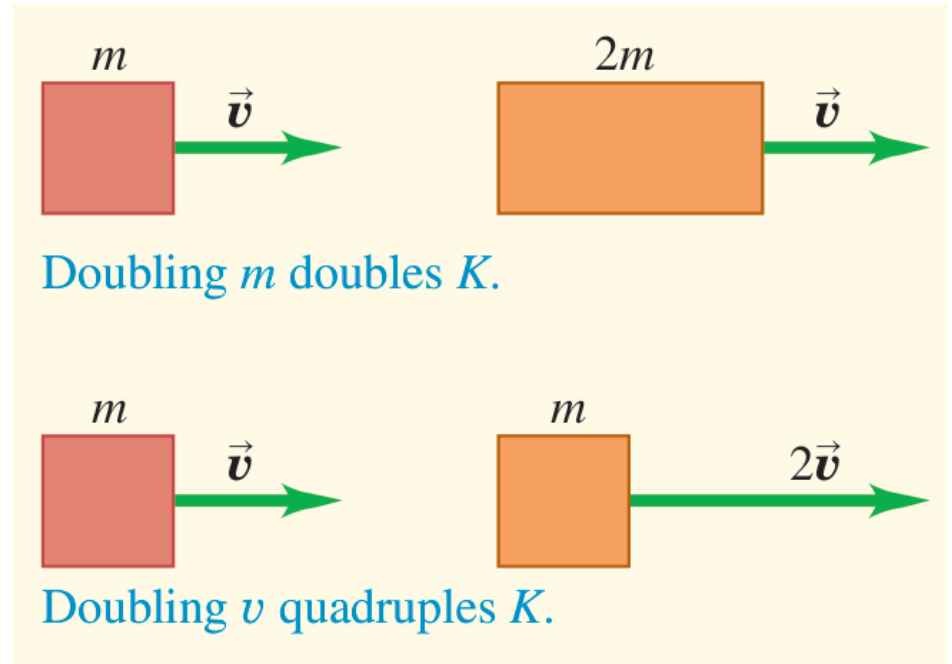
## Work done by a force

- The unit of work in SI units is  $1 \text{ J} = 1 \text{ N m}$
- Work is a scalar quantity
  - It can be positive or negative, but it has no direction in space
- See Examples 6.1 and 6.2

# Kinetic energy

- The kinetic energy  $K$  of a particle equals the amount of work required to accelerate the particle from rest to speed  $v$

$$K = \frac{1}{2}mv^2$$



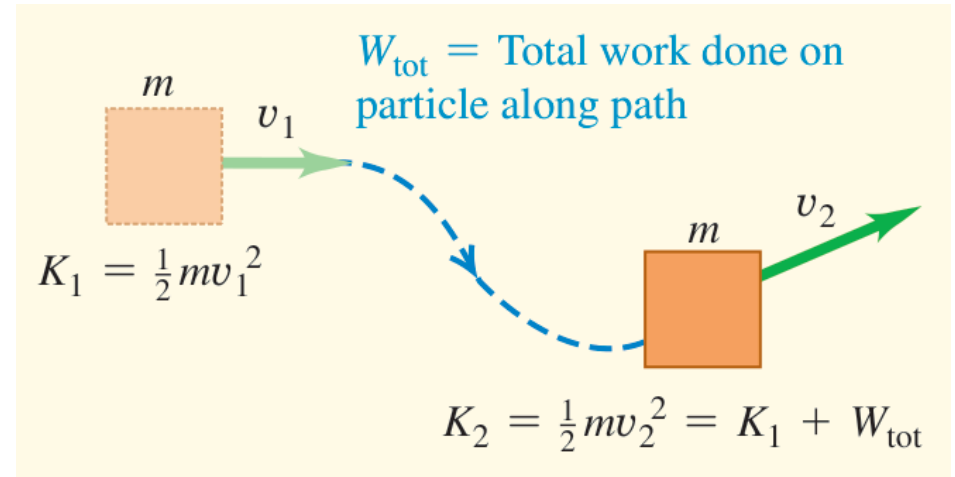
# Kinetic energy

- It is also equal to the amount of work the particle can do in the process of being brought to rest
- Kinetic energy is a scalar that has no direction in space; it is always positive or zero
- Its units are the same as the units of work:  
 $1 \text{ J} = 1 \text{ Nm} = 1 \text{ kg m}^2/\text{s}^2$

# Work-energy theorem

- When forces act on a particle while it undergoes a displacement, the particle's kinetic energy changes by an amount equal to the total work done on the particle by all the forces

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$



## Work-energy theorem

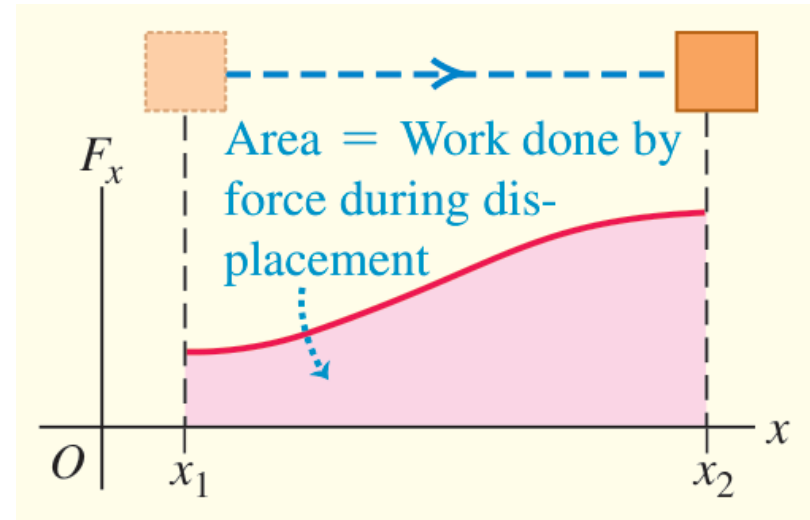
- This relationship, called the work-energy theorem, is valid whether the forces are constant or varying and whether the particle moves along a straight or curved path
- It is applicable only to objects that can be treated as particles
- See Examples 6.3–6.5

# Work done by a varying force or on curved path

- When a force varies during a straight-line displacement, the work done by the force is given by an integral

$$W = \int_{x_1}^{x_2} F_x \, dx$$

- See Examples 6.6 and 6.7



## Work done by a varying force or on curved path

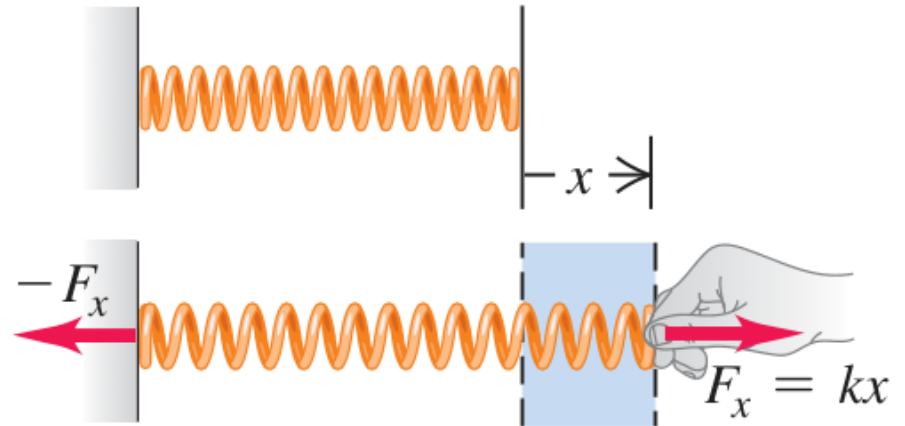
- When a particle follows a curved path, the work done on it by a force  $\vec{F}$  is given by an integral that involves the angle  $\phi$  between the force and the displacement

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l} = \int_{P_1}^{P_2} F \cos \phi \, dl = \int_{P_1}^{P_2} F_{\parallel} \, dl$$

- This expression is valid even if the force magnitude and the angle  $\phi$  vary during the displacement
- See Example 6.8

# Hooke's law

- To keep a spring stretched beyond its unstretched length by an amount  $x$ , we have to apply a force of equal magnitude at each end



## Hooke's law

- If the elongation  $x$  is not too great, the force we apply to right-hand end has an  $x$ -component directly proportional to  $x$  as in

$$F_x = kx$$

where  $k$  is a constant called the **force constant** (or spring constant) of the spring. The SI unit of  $k$  is N/m

- A floppy toy spring such as a slinky has a force constant of about 1 N/m. For the much stiffer springs in an automobile's suspension,  $k$  is about  $10^5$  N/m

- The observation that force is directly proportional to elongation for elongations that are not too great was made by Robert Hooke in 1678 and is known as **Hooke's law**
  - ▶ It really shouldn't be called a "law" since it's a statement about a specific device and not a fundamental law of nature
  - ▶ Real springs don't always obey this precisely, but it's still a useful idealized model

# Power

- Power is the time rate of doing work
- The average power  $P_{\text{av}}$  is the amount of work  $\Delta W$  done in time  $\Delta t$  divided by that time

$$P_{\text{av}} = \frac{\Delta W}{\Delta t}$$

$\Delta t = 5 \text{ s}$

Work you do on the box to lift it in  $\Delta t = 5 \text{ s}$ :  
 $\Delta W = 100 \text{ J}$

Your average power output:  
 $P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{100 \text{ J}}{5 \text{ s}}$   
 $= 20 \text{ W}$

$t = 0$

# Power

- The instantaneous power is the limit of the average power as  $\Delta t$  goes to zero

$$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

- When a force  $\vec{F}$  acts on a particle moving with velocity  $\vec{v}$ , the instantaneous power (the rate at which the force does work) is the scalar product of  $\vec{F}$  and  $\vec{v}$

$$P = \vec{F} \cdot \vec{v}$$

Work and kinetic energy [E6] &

- Like work and kinetic energy, power is a scalar quantity
- The SI unit of power is  $1 \text{ W} = 1 \text{ J/s}$
- See Examples 6.9 and 6.10

Potential energy and energy conservation



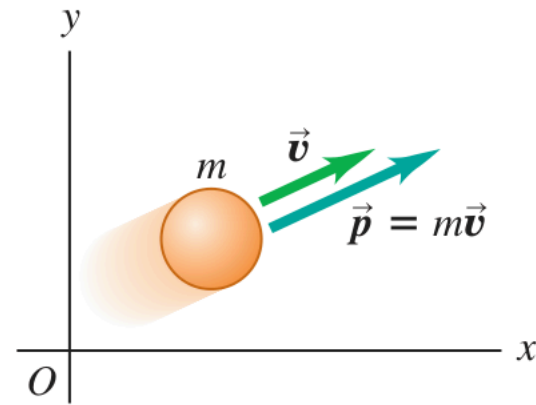
# Momentum, impulse, and collisions [E5] ✨



# Momentum of a particle

- The momentum  $\vec{p}$  of a particle is a vector quantity equal to the product of the particle's mass  $m$  and velocity  $\vec{v}$

$$\vec{p} = m\vec{v}$$



**Momentum  $\vec{p}$  is a vector quantity;** a particle's momentum has the same direction as its velocity  $\vec{v}$ .

# Momentum of a particle

- Newton's second law says that the net external force on a particle is equal to the rate of change of the particle's momentum

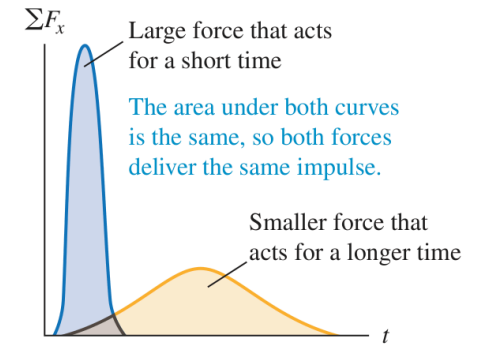
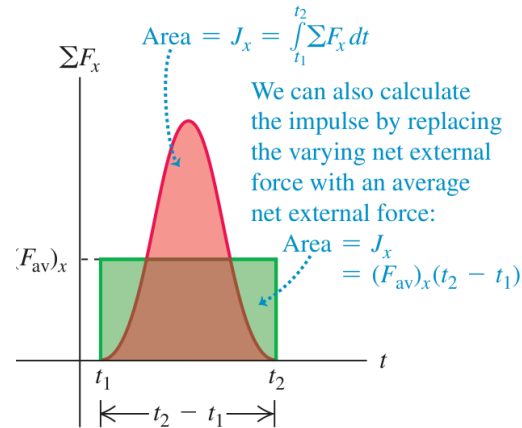
$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

# Impulse and momentum

- If a constant net external force  $\sum \vec{F}$  acts on a particle for a time interval  $\Delta t$  from  $t_1$  to  $t_2$ , the impulse  $\vec{J}$  of the net external force is the product of the net external force and the time interval

$$\vec{J} = \sum \vec{F}(t_2 - t_1) = \sum \vec{F} \Delta t$$

The area under the curve of net external force versus time equals the impulse of the net external force:



# Impulse and momentum

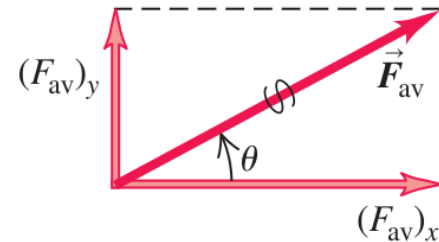
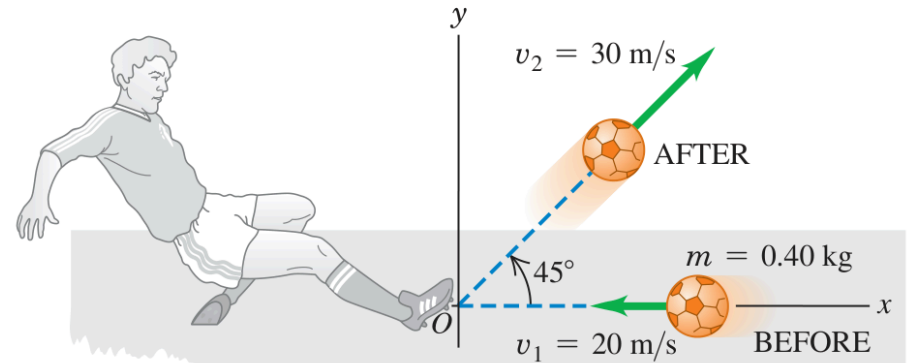
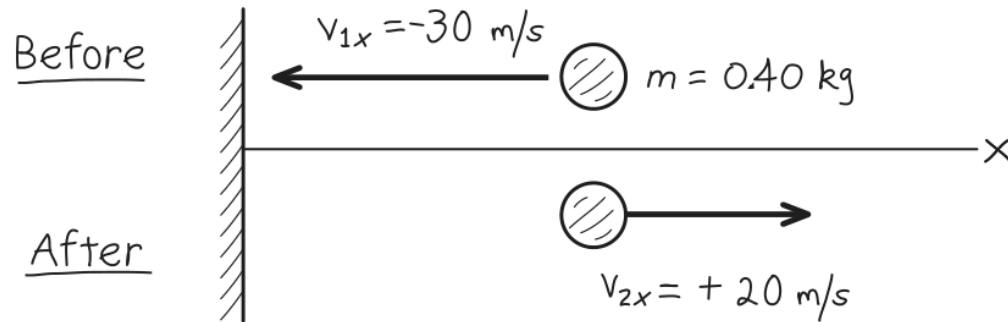
- If  $\sum \vec{F}$  varies with time,  $\vec{J}$  is the integral of the net external force over the time interval

$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} dt$$

- In any case, the change in a particle's momentum during a time interval equals the impulse of the net external force that acted on the particle during that interval

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

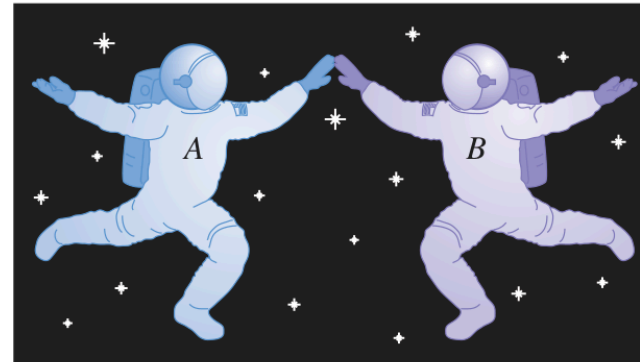
- The momentum of a particle equals the impulse that accelerated it from rest to its present speed
- See Examples 8.1–8.3



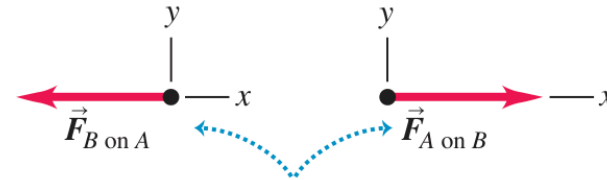
# Conservation of momentum

- An internal force is a force exerted by one part of a system on another
- An external force is a force exerted on any part of sys by something outside the system

$$\begin{aligned}\vec{P} &= \vec{p}_A + \vec{p}_B + \dots \\ &= m_A \vec{v}_A + m_B \vec{v}_B + \dots\end{aligned}$$



No external forces act on the two-astronaut system, so its total momentum is conserved.



The forces the astronauts exert on each other form an action–reaction pair.

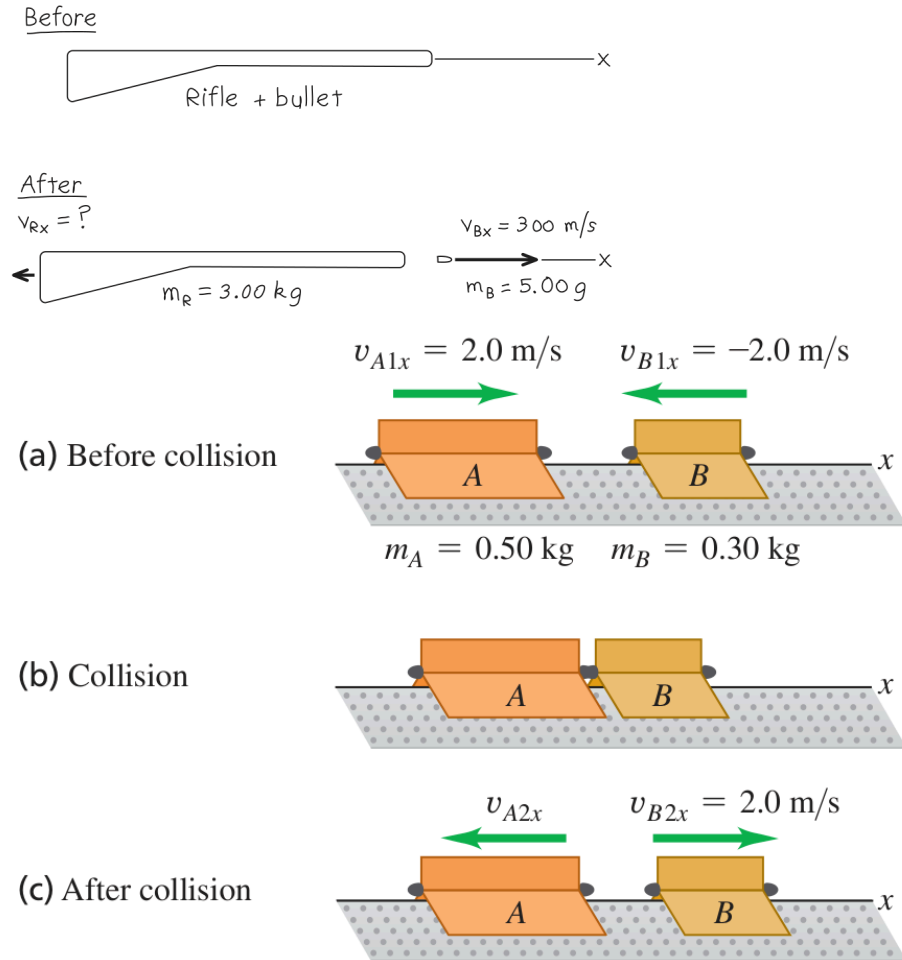
# Conservation of momentum

- If the net external force on a system is zero, the total momentum of the system  $\vec{P}$  (the vector sum of the momenta of the individual particles that make up the system) is constant, or conserved

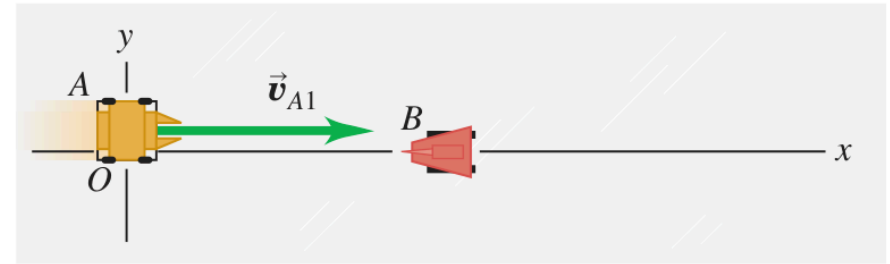
$$\sum \vec{F} = \vec{0} \implies \vec{P} = \text{constant}$$

- Each component of total momentum is separately conserved
- See Examples 8.4–8.6

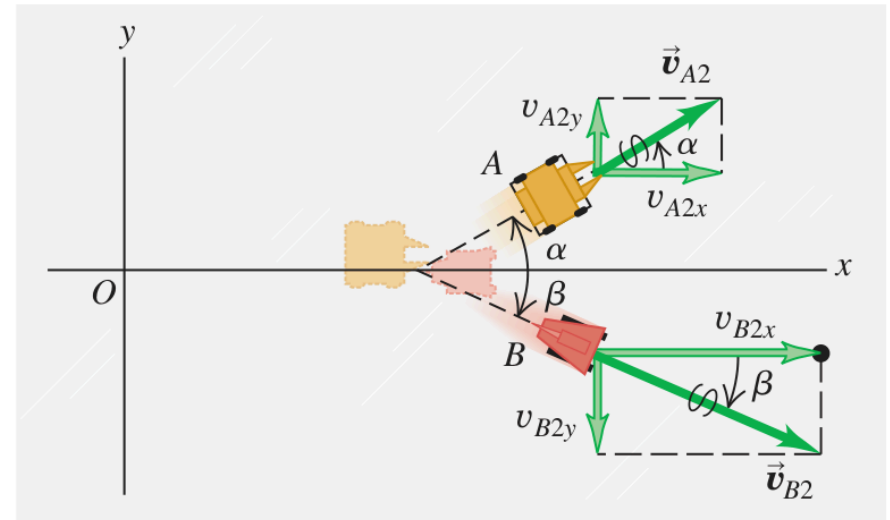
Momentum, impulse, and collisions [E5] 



(a) Before collision

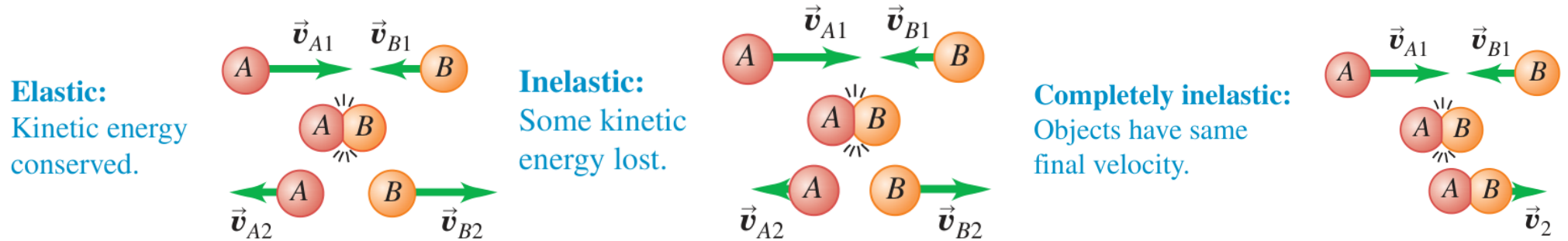


(b) After collision



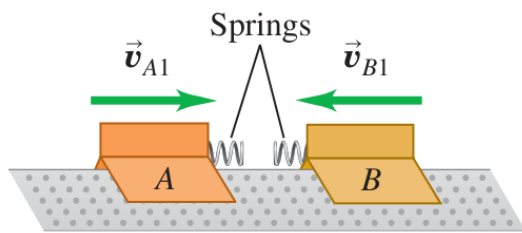
# Collisions

- In typical collisions, initial and final total momenta are equal

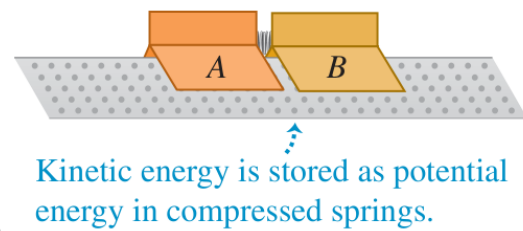


- In an elastic collision between two objects, the initial and final total kinetic energies are also equal, and the initial and final relative velocities have the same magnitude

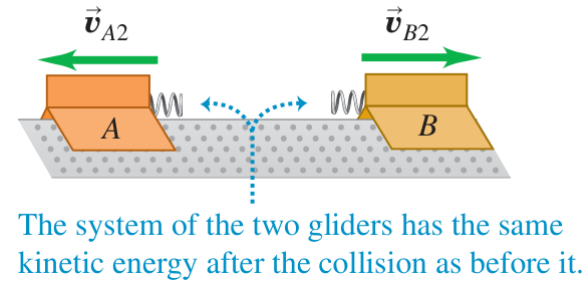
(a) Before collision



(b) Elastic collision

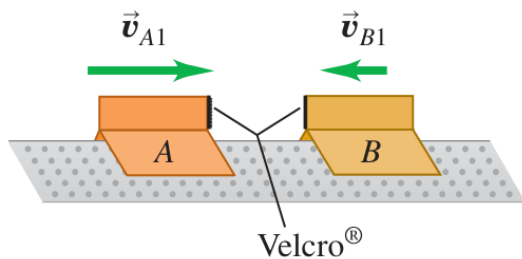


(c) After collision

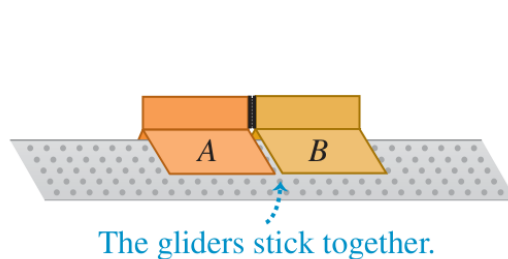


- In an inelastic two-object collision, the total kinetic energy is less after the collision than before

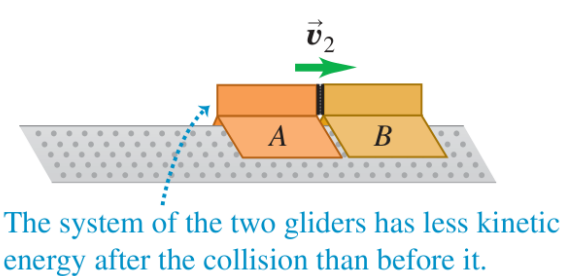
(a) Before collision



(b) Completely inelastic collision

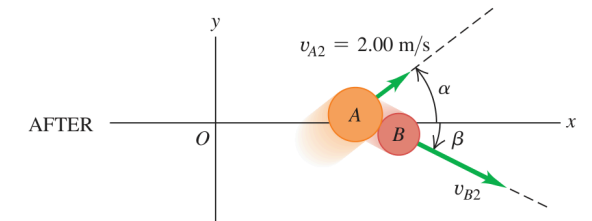
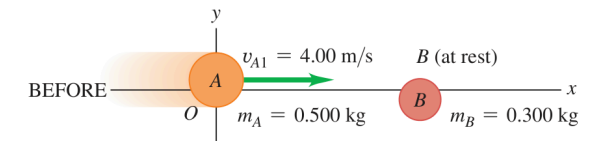
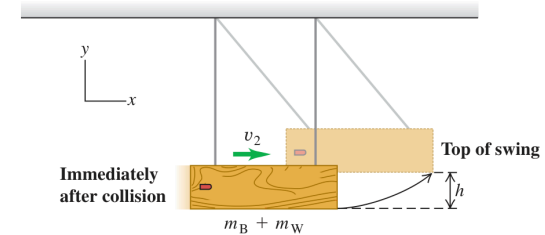
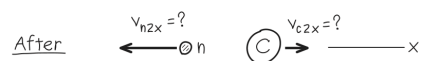
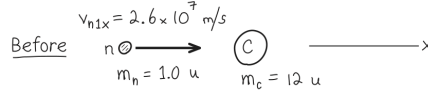
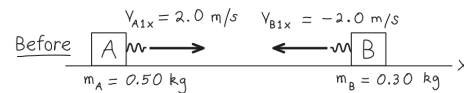
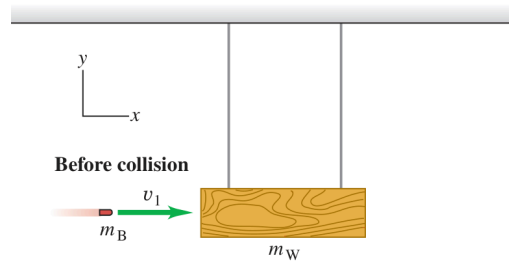
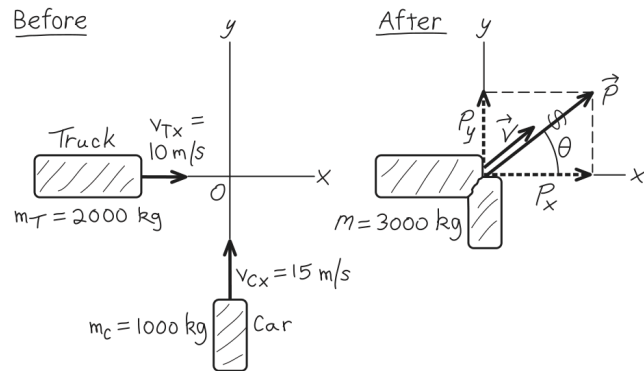
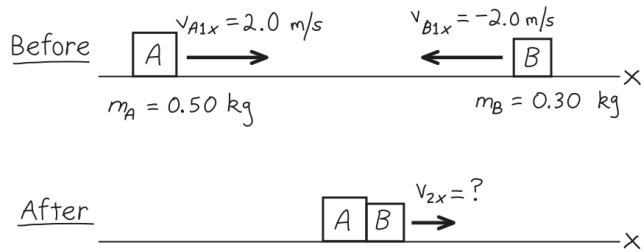


(c) After collision



Momentum, impulse, and collisions [E5] 

- If the two objects have the same final velocity, the collision is completely inelastic
- See Examples 8.7–8.12



## Center of mass

- The position vector of the center of mass of a system of particles  $\vec{r}_{\text{cm}}$  is a weighted average of the positions  $\vec{r}_1, \vec{r}_2, \dots$  of the individual particles

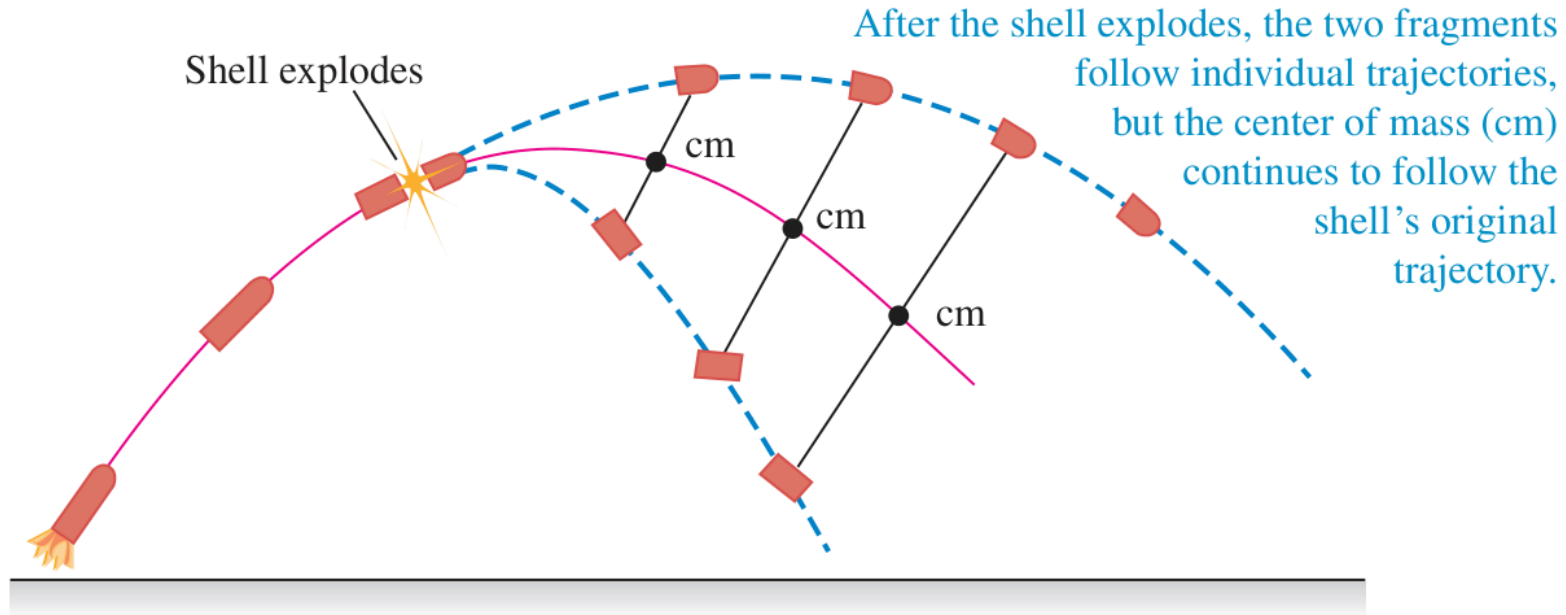
$$\vec{r}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

- The total momentum  $\vec{P}$  of a system equals the system's total mass  $M$  multiplied by the velocity of its center of mass  $\vec{v}_{\text{cm}}$

$$\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \dots = M \vec{v}_{\text{cm}}$$

- The center of mass moves as though all the mass  $M$  were concentrated at that point
- If the net external force on the system is zero, the center-of-mass velocity  $\vec{v}_{\text{cm}}$  is constant
- If the net external force is not zero, the center of mass accelerates as though it were a particle of mass  $M$  being acted on by the same net external force

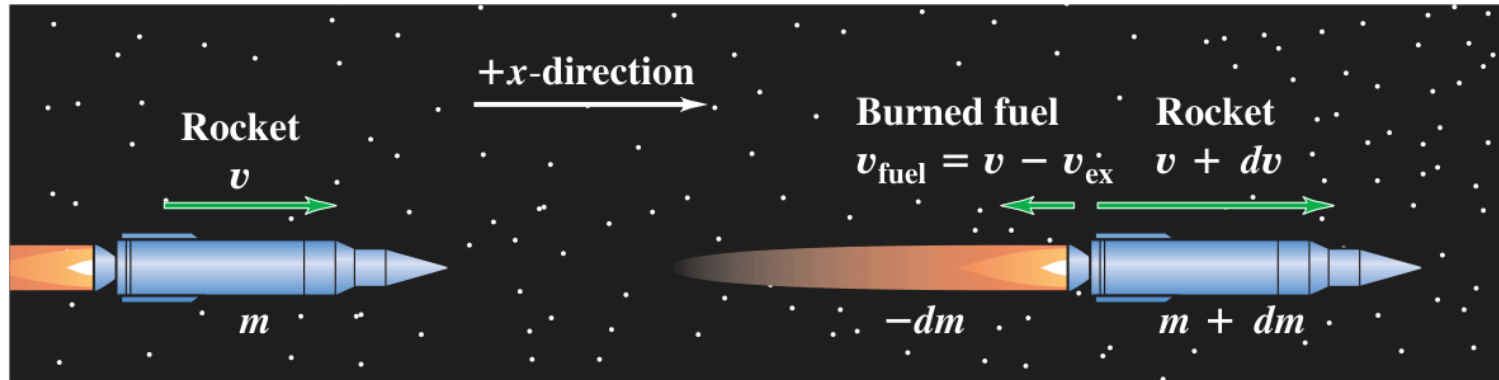
$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$$



- See Examples 8.13 and 8.14

# Rocket propulsion

- In rocket propulsion, the mass of a rocket changes as the fuel is used up and ejected from the rocket
- Analysis of the motion of the rocket must include the momentum carried away by the spent fuel as well as the momentum of the rocket itself



At time  $t$ , the rocket has mass  $m$  and  $x$ -component of velocity  $v$ .

At time  $t + dt$ , the rocket has mass  $m + dm$  (where  $dm$  is inherently *negative*) and  $x$ -component of velocity  $v + dv$ . The burned fuel has  $x$ -component of velocity  $v_{\text{fuel}} = v - v_{\text{ex}}$  and mass  $-dm$ . (The minus sign is needed to make  $-dm$  *positive* because  $dm$  is negative.)

- See Examples 8.15 and 8.16

Rotation of rigid bodies ●

Dynamics of rotational motion ●

**Equilibrium and elasticity** ●

# Fluid mechanics [E7] 🌊

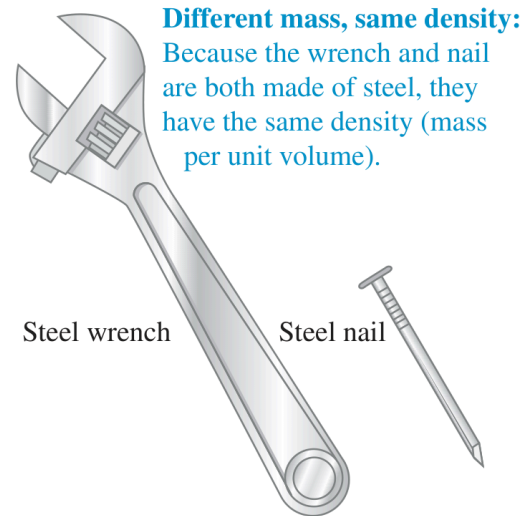


# Density and pressure

- Density is mass per unit volume

$$\rho = \frac{m}{V}$$

- If a mass  $m$  of homogeneous material has volume  $V$ , its density  $\rho$  is the ratio  $m/V$



# Density and pressure

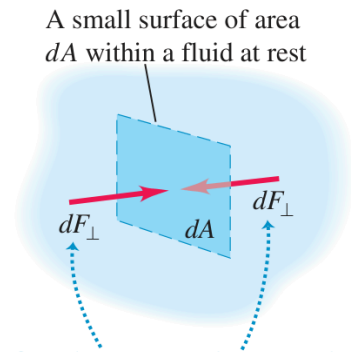
- Specific gravity is the ratio of the density of a material to the density of water
- See Example 12.1

# Density and pressure

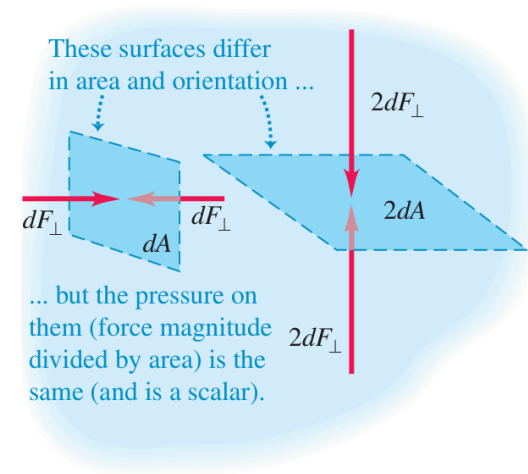
- Pressure is normal force per unit area

$$p = \frac{dF_{\perp}}{dA}$$

- Absolute pressure is the total pressure in a fluid
- Gauge pressure is difference between absolute pressure and atmospheric pressure

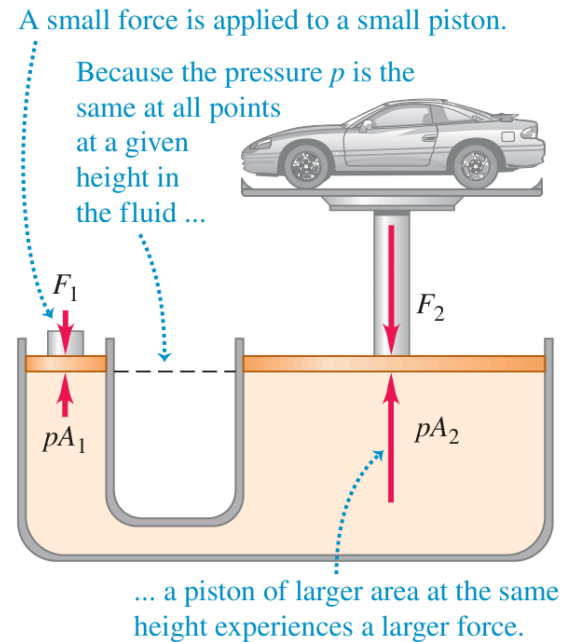


The surface does not accelerate, so the surrounding fluid exerts equal normal forces on both sides of it. (The fluid cannot exert any force parallel to the surface, since that would cause the surface to accelerate.)



# Density and pressure

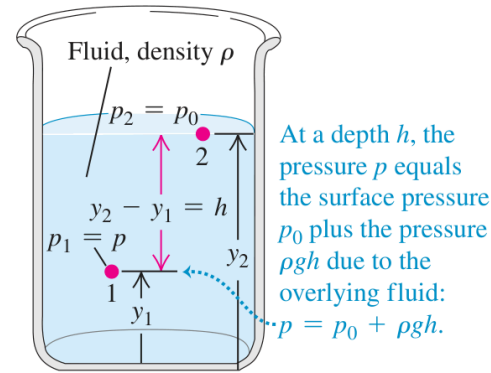
- Pascal's law states that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid
- The SI unit of pressure is the pascal (Pa):  $1 \text{ Pa} = 1 \text{ N/m}^2$
- See Example 12.2



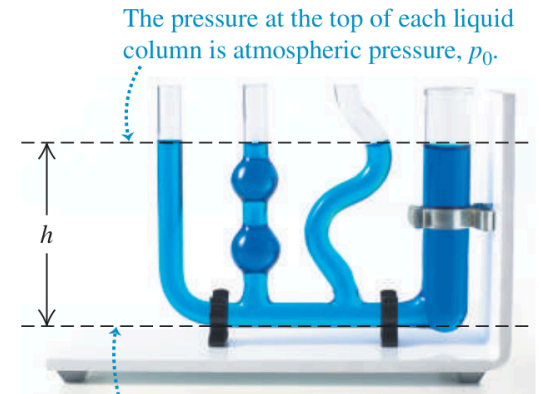
# Pressures in a fluid at rest

- The pressure difference between points 1 and 2 in a static fluid of uniform density  $\rho$  (an incompressible fluid) is proportional to the difference between elevations  $y_1$  and  $y_2$

$$p_2 - p_1 = -\rho g(y_2 - y_1)$$



Pressure difference between levels 1 and 2:  
 $p_2 - p_1 = -\rho g(y_2 - y_1)$   
 The pressure is greater at the lower level.



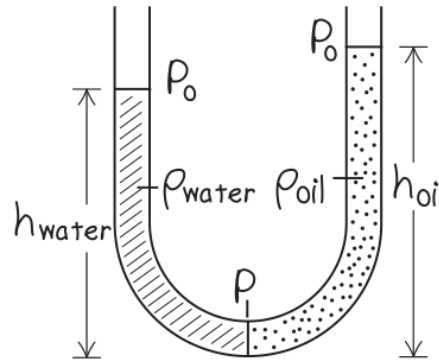
The pressure at the bottom of each liquid column has the same value  $p$ .  
 The difference between  $p$  and  $p_0$  is  $\rho gh$ , where  $h$  is the distance from the top to the bottom of the liquid column. Hence all columns have the same height.

# Pressures in a fluid at rest

- If the pressure at surface of an incompressible liquid at rest is  $p_0$ , then the pressure at a depth  $h$  is greater by an amount  $\rho gh$

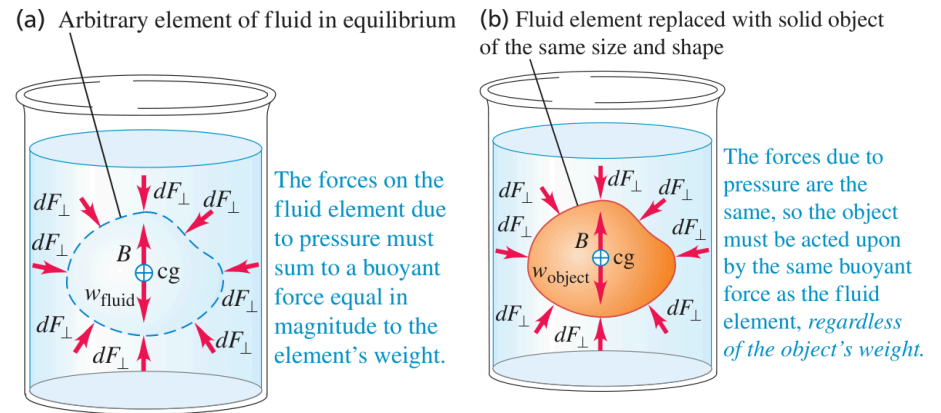
$$p = p_0 + \rho gh$$

- See Examples 12.3 and 12.4



# Buoyancy

- Archimedes's principle states that when object is immersed in a fluid, the fluid exerts an upward buoyant force on the object equal to the weight of fluid that the object displaces
- Buoyant force is upward and equal in magnitude to weight
- See Example 12.5

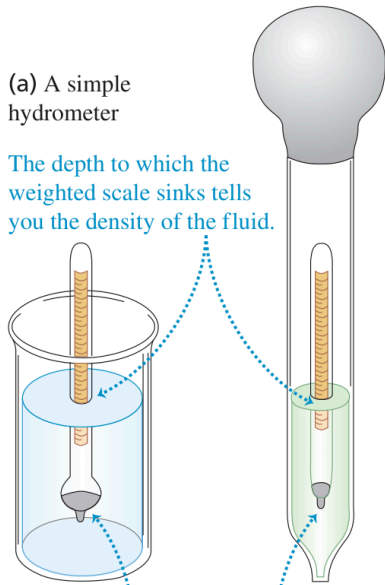


# Buoyancy

(b) Using a hydrometer to measure the density of battery acid or antifreeze

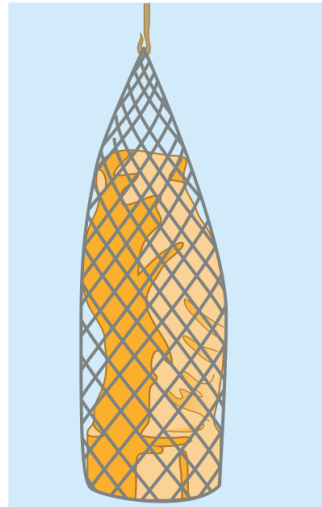
(a) A simple hydrometer

The depth to which the weighted scale sinks tells you the density of the fluid.

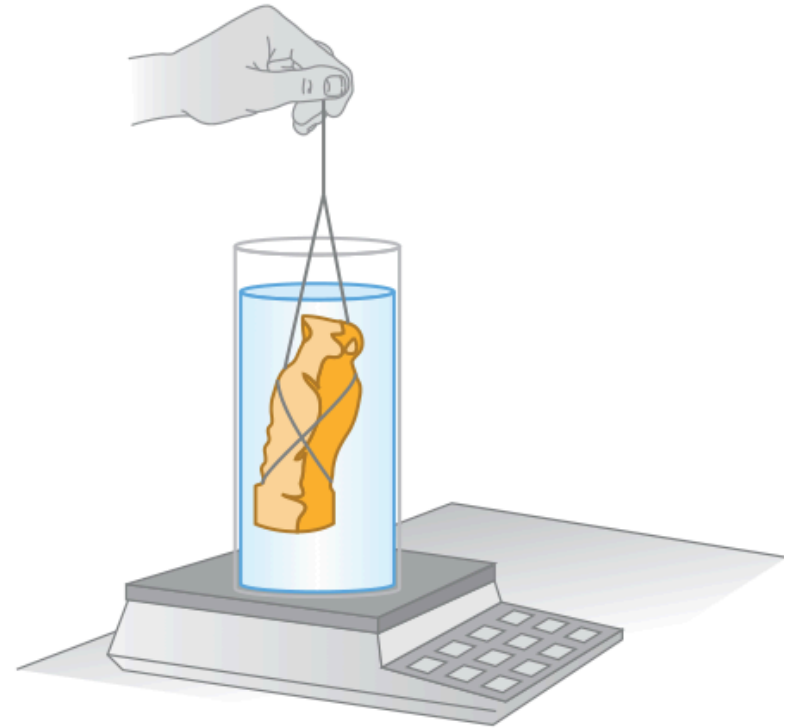
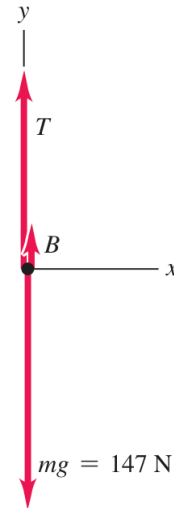





The weight at the bottom makes the scale float upright.

(a) Immersed statue in equilibrium



(b) Free-body diagram of statue



- The surface of the liquid behaves like a membrane under tension
- Surface tension arises because the molecules of the liquid exert attractive forces on each other
  - ▶ eg.  resting atop a water surface even though its density is several times that of water
  - ▶ explains why raindrops are spherical, not teardrop-shaped 
  - ▶ explains why hot, soapy water is used for washing 

Molecules in a liquid are attracted by neighboring molecules.

At the surface, the unbalanced attractions cause the surface to resist being stretched.

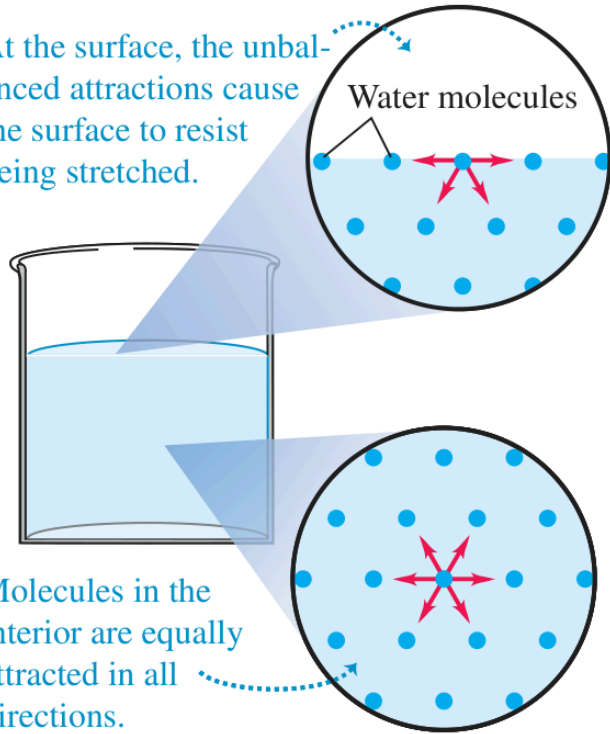
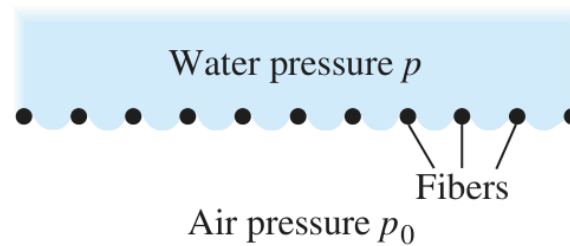
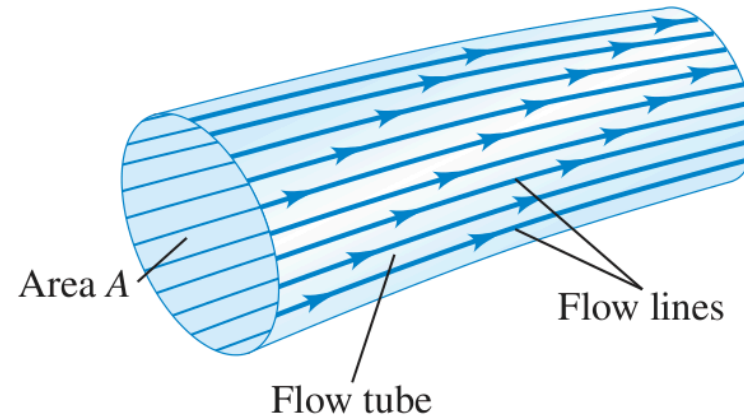


Figure 12.16 Surface tension makes it difficult to force water through small crevices. The required water pressure  $p$  can be reduced by using hot, soapy water, which has less surface tension.



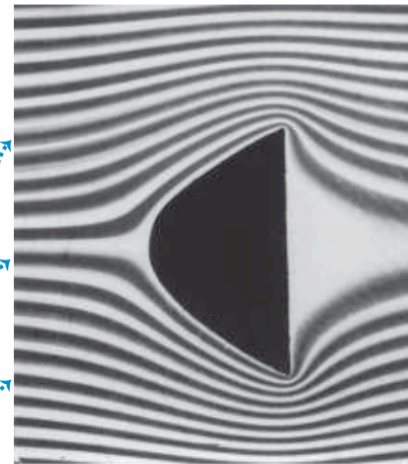
# Fluid flow

- An ideal fluid is incompressible and has no viscosity (no internal friction)
- A flow line is the path of a fluid particle. A streamline is a curve tangent at each point to velocity at that point. A flow tube is a tube bounded at its sides by flow lines

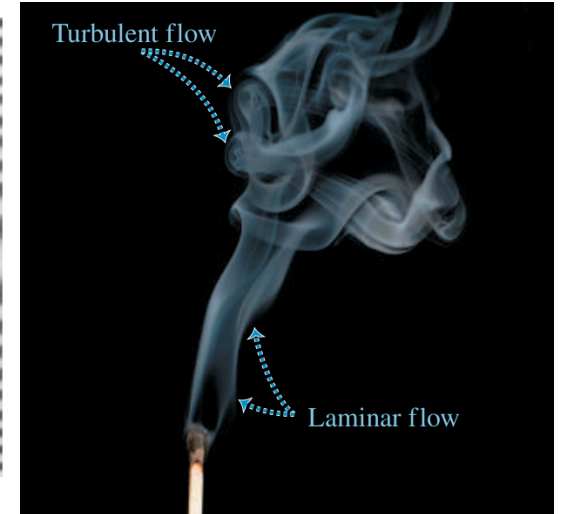


# Fluid flow

- In laminar flow, layers of fluid slide smoothly past each other
- In turbulent flow, there is great disorder and a constantly changing flow pattern



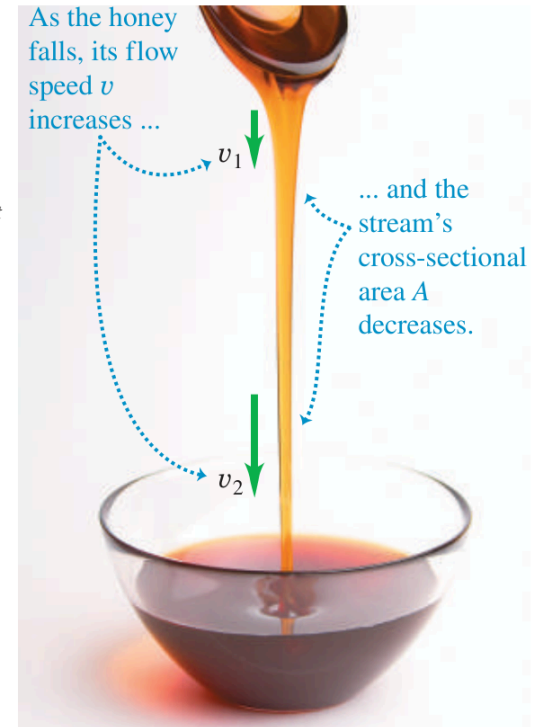
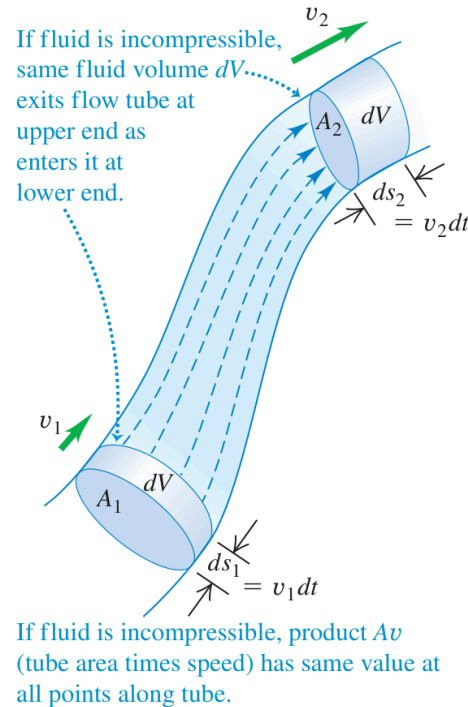
Dark-colored dye follows streamlines of laminar flow (flow is from left to right).



# Fluid flow

- Conservation of mass in an incompressible fluid is expressed by the continuity equation, which relates the flow speeds  $v_1$  and  $v_2$  for two cross sections  $A_1$  and  $A_2$  in a flow tube

$$A_1 v_1 = A_2 v_2$$



The volume flow rate  $dV/dt = Av$  remains constant.

# Fluid flow

- The product  $Av$  equals the volume flow rate,  $dV/dt$ , the rate at which volume crosses a section of the tube

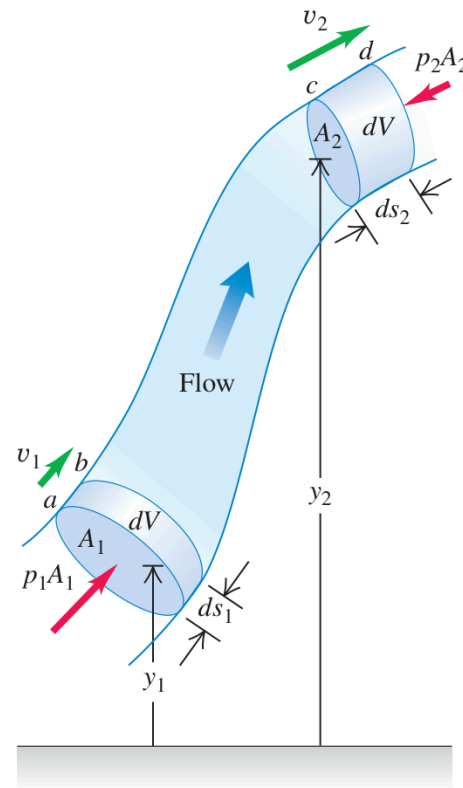
$$\frac{dV}{dt} = Av$$

- See Example 12.6

# Fluid flow

- Bernoulli's equation states that a quantity involving the pressure  $p$ , flow speed  $v$ , and elevation  $y$  has the same value anywhere in a flow tube, assuming steady flow in an ideal fluid

$$p + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$$

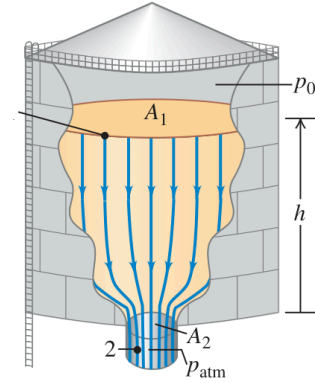
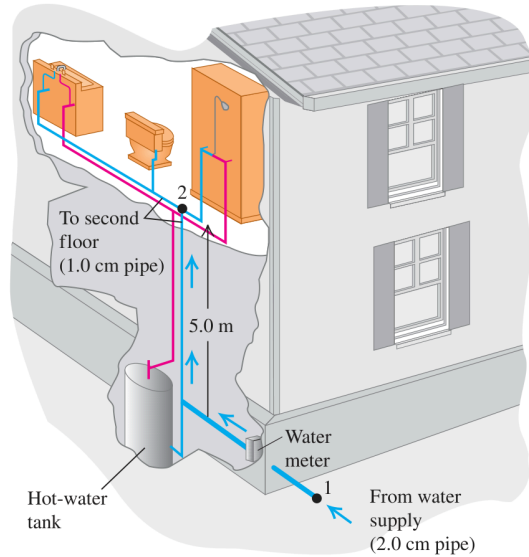


# Fluid flow

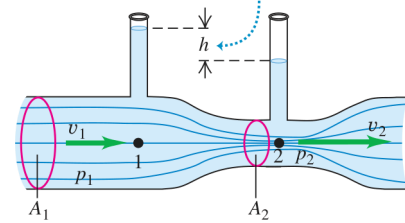
- This equation can be used to relate the properties of the flow at any two points
- See Examples 12.7–12.10

**BIO APPLICATION Why Healthy Giraffes Have High Blood Pressure**

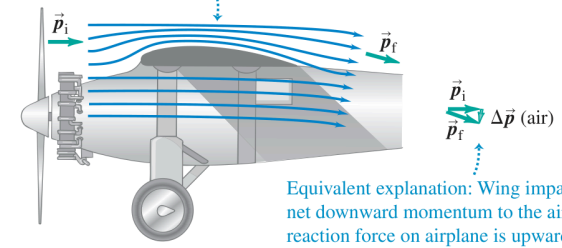
Bernoulli's equation suggests that as blood flows upward at roughly constant speed  $v$  from the heart to the brain, the pressure  $p$  will drop as the blood's height  $y$  increases. For blood to reach the brain with the required minimal pressure, the human heart provides a maximum (systolic) gauge pressure of about 120 mm Hg. The vertical distance from heart to brain is much larger for a giraffe, so its heart must produce a much greater maximum gauge pressure (about 280 mm Hg).



Difference in height results from reduced pressure in throat (point 2).



Flow lines are crowded together above the wing, so flow speed is higher there and pressure is lower.



Equivalent explanation: Wing imparts a net downward momentum to the air, so reaction force on airplane is upward.

Gravitation ●

# Periodic motion [E6] &



**Mechanical waves**



Sound and hearing ●

Temperature and heat ●

**Thermal properties of matter ●**

**The first law of thermodynamics**



**The second law of thermodynamics ●**

**Electric charge and electric field** ●

Gauss's law ●

**Electric potential** ●

Capacitance and dielectrics ●

**Current, resistance, and electromotive force [E8]**





# Direct-current circuits [E8]



# Resistors in series and parallel

- When several resistors  $R_1, R_2, R_3, \dots$  are connected in series, the equivalent resistance  $R_{\text{eq}}$  is the sum of the individual resistances
- The same current flows through all the resistors in a series connection
- When several resistors are connected in parallel, the reciprocal of equivalent resistance  $R_{\text{eq}}$  is the sum of the reciprocals of the individual resistances
- All resistors in a parallel connection have the same potential difference between their terminals

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \dots \quad (\text{resistors in series})$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \quad (\text{resistors in parallel})$$

# Kirchhoff's rules

- Kirchhoff's junction rule is based on conservation of charge
- It states that the algebraic sum of the currents into any junction must be zero
- Kirchhoff's loop rule is based on conservation of energy and the conservative nature of electrostatic fields
- It states that the algebraic sum of potential differences around any loop must be zero
- Careful use of consistent sign rules is essential in applying Kirchhoff's rules

- See Examples 26.3–26.7

$$\sum I = 0 \quad (\text{junction rule})$$

$$\sum V = 0 \quad (\text{loop rule})$$

# Electrical measuring instruments

- In a d'Arsonval galvanometer, the deflection is proportional to the current in the coil
- For a larger current range, a shunt resistor is added, so some of the current bypasses the meter coil
- Such an instrument is called an ammeter
- If the coil and any additional series resistance included obey Ohm's law, the meter can also be calibrated to read potential difference or voltage
- The instrument is then called a voltmeter

- A good ammeter has very low resistance; a good voltmeter has very high resistance
- See Examples 26.8–26.11

## *R-C* circuits

- When a capacitor is charged by a battery in series with a resistor, the current and capacitor charge are not constant
- The charge approaches its final value asymptotically and the current approaches zero asymptotically
- The charge and current in the circuit are given by Eqs. (26.12) and (26.13)
- After a time  $t = RC$ , the charge has approached within  $1/e$  of its final value

- This time is called the time constant or relaxation time of the circuit
- When the capacitor discharges, the charge and current are given as functions of time by Eqs. (26.16) and (26.17)
- The time constant is the same for charging and discharging
- See Examples 26.12 and 26.13

$$\begin{aligned}q &= C\varepsilon(1 - e^{-t/RC}) \\ &= Q_f(1 - e^{-t/RC})\end{aligned}$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

# Household wiring

- In household wiring systems, the various electrical devices are connected in parallel across the power line, which consists of a pair of conductors, one “hot” and the other “neutral”
- An additional “ground” wire is included for safety
- The maximum permissible current in a circuit is determined by the size of the wires and the maximum temperature they can tolerate
- Protection against excessive current and the resulting fire hazard is provided by fuses or circuit breakers

- See Example 26.14

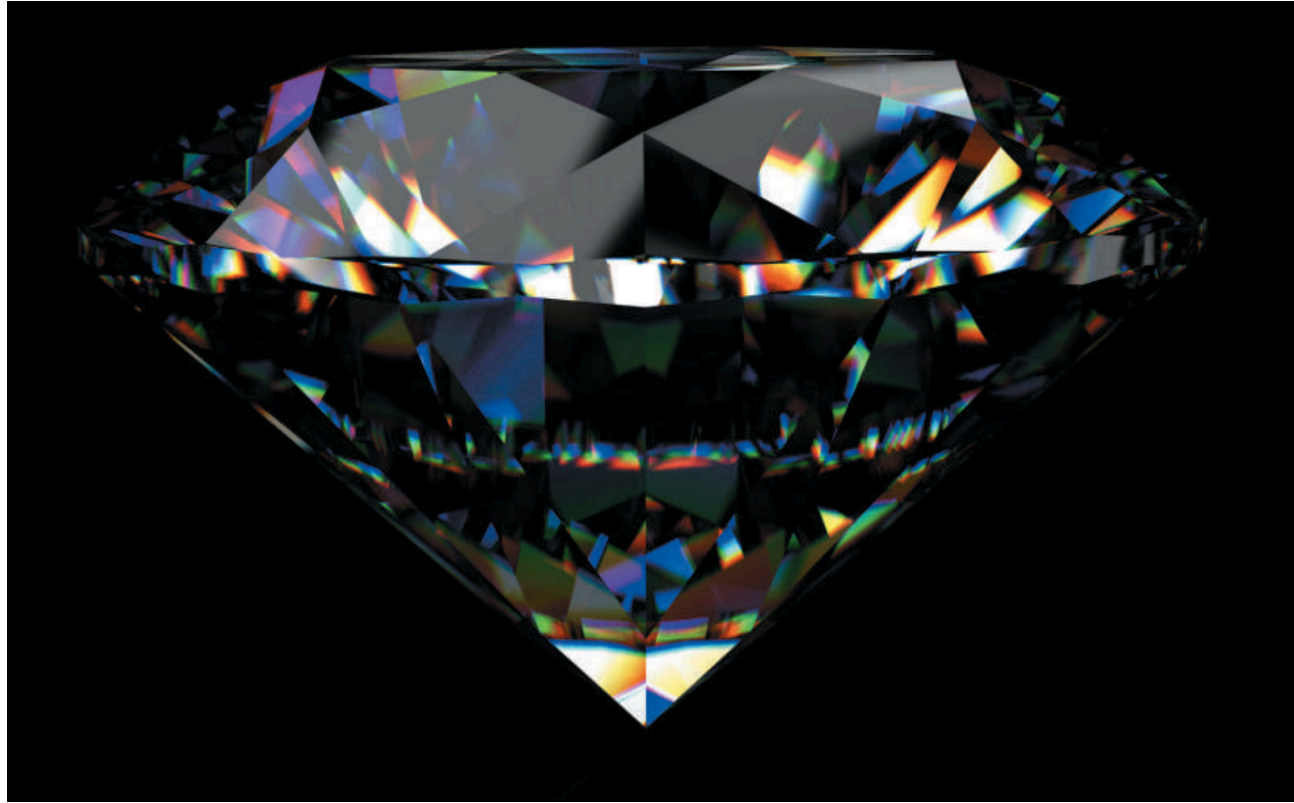
**Magnetic field and magnetic forces**



Sources of magnetic field ●

**Electromagnetic induction field ●**

# The nature and propagation of light [E10]

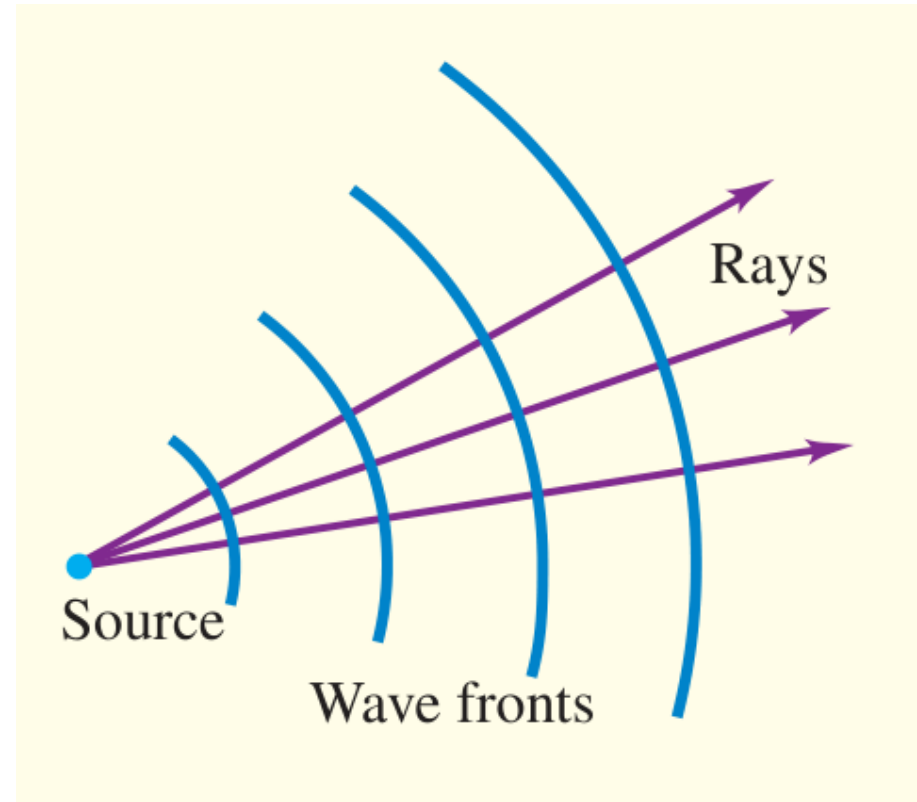


# Light and its properties

- Light is an electromagnetic wave
  - ▶ But when emitted or absorbed, it also shows particle properties
  - ▶ This is what's known as the **dual nature of light**
  - ▶ It is emitted by accelerated electric charges

# Light and its properties

- A **wave front** is a surface of constant phase
- Wave fronts move with a speed equal to the propagation speed of wave
- A **ray** is a line along the direction of propagation, perpendicular to wave fronts



# Light and its properties

- When light is transmitted from one material to another, its frequency is unchanged, but wavelength/wavespeed can change
- The **index of refraction**  $n$  of a material is the ratio of the speed of light in vacuum  $c$  to the speed  $v$  in the material

$$n = c/v$$

- If  $\lambda_0$  is the wavelength in vacuum, the same wave has a shorter wavelength  $\lambda$  in a medium with index of refraction  $n$

$$\lambda = \lambda_0/n$$

# Reflection and refraction

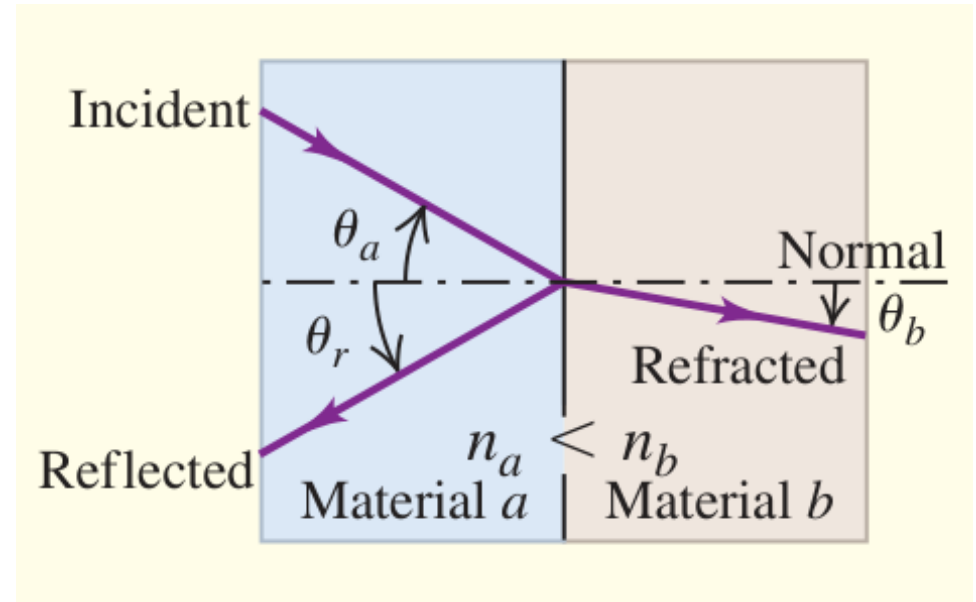
- At a smooth interface between two optical materials, the incident, reflected, and refracted rays and the normal to the interface all lie in a single plane called the **plane of incidence**
- See Examples 33.1 and 33.3

- The **law of reflection** states that the angles of incidence and reflection are equal

$$\theta_r = \theta_a$$

- The **law of refraction** relates the angles of incidence and refraction to the indexes of refraction of the materials

$$n_a \sin \theta_a = n_b \sin \theta_b$$



# Total internal reflection

- When a ray travels in a material of index of refraction  $n_a$  toward a material of index  $n_b < n_a$ , **total internal reflection** occurs at interface when the angle of incidence equals or exceeds a critical angle  $\theta_{\text{crit}}$ :

$$\sin \theta_{\text{crit}} = n_b / n_a$$

