

Phys 20.01 lecture notes

R. Torres
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These lecture notes for Phys 20.01 (Elementary Physics I) offer a deeper dive into the topics than the class slides. They will be updated regularly throughout the course, drawing from

- P.P. Urone and R. Hinrichs, College Physics 2e, OpenStax (2022), [open access](#)
- P.G. Hewitt, Conceptual Physics 13e, Pearson (2021)
- H.D. Young and R.A. Freedman, University Physics with Modern Physics 15e, Pearson (2019).

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1. Kinematics

This module explores one and two dimensional kinematics, specifically on

- displacement, velocity, acceleration (2.1–2.4)
- motion with constant acceleration, falling objects (2.5, 2.7)
- vectors, motion in two dimensions (3.1–3.3)
- projectile motion, velocity addition (3.4, 3.5)
- problem solving, graphical analysis (2.6, 2.8) ★.

1.1. One-dimensional kinematics

- Motion is everywhere we look
 - Obvious: tennis game, GPS satellites
 - Subtle: blood flowing in your veins despite you resting, molecules vibrating within an inanimate table
- **Kinematics** is the study of motion without considering its causes
 - Usually, causes = forces
 - **Dynamics** is such study with causes
- **Position** is where an object is at any particular time
 - More precisely, where is the object relative to a convenient reference frame

- ▶ Reference frame is a perspective from which you're making observations and is composed of an origin, a set of axes, and a clock. You can imagine said frame is a coordinate system you choose to describe spacetime
- ▶ Often, we take another stationary object in said frame eg. rocket launch relative to earth
- **Displacement** Δx is the change in position of an object. In symbols,

$$\Delta x = x_f - x_0$$

where x_0 and x_f are the initial and final positions

- ▶ Btw, Δx means change in x , and x_0 is read as "x naught"
- ▶ Object moves relative to a reference frame \Rightarrow object's position changes \Rightarrow object has been "displaced" \Rightarrow displacement
- ▶ Its SI unit is meter (m), but feel free to use km, mi, ft, etc. as long as you convert properly and use such units consistently
- ▶ It is a vector quantity \Rightarrow has magnitude and direction
- ▶ Choose which direction is positive, usually it is the rightward or upward direction
- **Distance traveled** is the total length of the path traveled between two positions
 - ▶ Distance is the magnitude of displacement between two positions
 - ▶ Recall that displacement only considers the initial and final positions
 - ▶ Thus, distance traveled between two positions is not necessarily the same as the distance between them (that is the magnitude of displacement). They're equal if path is straight, otherwise former can be greater than the latter
 - ▶ Both are scalar quantities \Rightarrow only magnitude and no direction (and parity \pm)
 - ▶ In kinematics, we almost always deal with displacement and distance, not distance traveled
- A **vector** is any quantity with both magnitude and direction eg. displacement, velocity, force
 - ▶ A **scalar** is any quantity that has magnitude but no direction eg. distance, temperature, energy, speed, height
 - ▶ In 1-dim, the direction of a vector is represented simply by parity \pm
 - ▶ Generally and graphically, it is represented by arrows. The arrow has a length proportional to vector's magnitude and points in the vector's direction
 - ▶ Scalars can be negative but parity indicates a point on a scale rather than a direction
- A **coordinate system** must be assigned in a reference frame in order to describe a vector
 - ▶ In 1-dim, positive is typically the rightward direction for horizontal motion and upward for vertical motion
 - ▶ The opposite direction can be chosen if more convenient, just be consistent throughout
- **Time** is measured in terms of change and is the interval over which change occurs. Elapsed time Δt for an event of change is

$$\Delta t = t_f - t_0$$

where t_0 and t_f are times at the beginning and end of event

- ▶ Its SI unit is second (s)
- ▶ For convenience and by convention, initial time t_0 is often taken to be zero ($t_0 = 0$) as if measured with a stopwatch. Elapsed time is just then $\Delta t = t_f \equiv t$
- **Velocity**, specifically average velocity \bar{v} , is defined as displacement divided by travel time. In symbols,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}$$

- ▶ Its SI unit is m/s, and it is a vector since displacement is a vector
- ▶ It only considers the initial and final positions, providing no info about the motion in between. By considering smaller segments of the motion over shorter time intervals, we obtain more info. This is analogous to the derivative in calculus which considers infinitesimally small interval
- ▶ Instantaneous velocity v is the velocity at a specific instant of time, or is the average velocity for an infinitesimal interval, as in

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- ▶ Instantaneous speed is the magnitude of the instantaneous velocity
- ▶ Average speed is the total distance traveled divided by travel time
- ▶ It is not the magnitude of average velocity since velocity makes use of displacement, not distance traveled
- ▶ Speed, be it average or instantaneous, is a scalar
- **Acceleration**, specifically average acceleration \bar{a} is the rate at which velocity changes. In symbols,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$$

- ▶ It is a vector, and its SI unit is m/s²
- ▶ It can be caused by a change in the velocity's magnitude, direction, or both
- ▶ Instantaneous acceleration a is the acceleration at a specific instant of time

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- ▶ While acceleration is in the direction of the change in velocity, it is not always in the direction of an object's motion
- ▶ Deceleration is an acceleration whose direction is opposite to that of the velocity, resulting in a decrease in velocity. Simply put, it acts in the opposite direction to the object's motion, causing it to slow down
- ▶ Negative acceleration is acceleration in the negative direction in the chosen coordinate system, and is not necessarily deceleration. That is, it does not necessarily cause an object to slow down
- In 1-dim, the **kinematic equations** for motion with **constant a** are

$$x = x_0 + \bar{v}t, \quad \bar{v} = \frac{1}{2}(v_0 + v), \quad v = v_0 + at,$$

$$x = x_0 + v_0t + \frac{1}{2}at^2, \quad v^2 = v_0^2 + 2a(x - x_0)$$

- ▶ Here, we take acceleration to be constant so $\bar{a} = a$ at all times, and initial time to be zero
- ▶ In vertical motion, y takes the place of x
- An object in **free-fall** experiences constant a if air resistance is negligible
 - ▶ Free-fall is the state of motion resulting from only gravitational force
 - ▶ On earth, all free-falling objects have an acceleration a_g due to gravity, which averages at $a_g = 9.8\text{m/s}^2$
 - ▶ Acceleration a_g can be taken either as $+a_g$ or $-a_g$ depending on your choice of coordinate system. If you choose upward to be positive, $a = -a_g$ is negative, otherwise $a = a_g$. Former is the typical choice

- ▶ Since a is constant in free-fall, you can use above kinematic equations where either $a = \pm a_g$

1.2. Two-dimensional kinematics

- In 2-dim, the shortest path between any two points is a straight line
 - ▶ This path can be represented by a vector with **horizontal and vertical components**
 - ▶ These components are independent of one another. Relating to kinematics, motion in the horizontal direction does not affect motion in the vertical, and vice versa
- **Vectors** quantities have magnitude and direction, and combine as per rules of vector addition
 - ▶ Scalar quantities are just magnitudes and combine as per usual rules of arithmetic
 - ▶ The negative of a vector has the same magnitude but points in the opposite direction, eg. deceleration
 - ▶ Graphically, adding vectors \vec{A} and \vec{B} results in the resultant vector $\vec{R} = \vec{A} + \vec{B}$ obtained by placing the tail of \vec{B} at the head of \vec{A} and drawing \vec{R} from the tail of \vec{A} to the head of \vec{B} , or the other way around
 - ▶ Vectors can be added by using components of vectors. The x -component of $\vec{R} = \vec{A} + \vec{B}$ is the sum of the x -components of \vec{A} and \vec{B} , and likewise for y (and even z), as in

$$R_x = A_x + B_x, \quad R_y = A_y + B_y, \quad R_z = A_z + B_z$$

- TODO: graphical methods of vector addition
- TODO: analytical methods of vector addition
- An object is in **projectile motion** through the air if it is subject only to gravitational acceleration a_g . To solve problems involving this motion, try to perform the following
 - ▶ Determine the coordinate system
 - ▶ Analyze the horizontal component using

$$a_x = 0, \quad x = x_0 + v_x t, \quad v_x = v_{0x} \text{ (constant velocity)}$$

- ▶ Analyze the vertical component using

$$a_y = -a_g = -9.8 \text{ m/s}^2, \quad y = y_0 + \frac{1}{2}(v_{0y}t + v_y)t, \quad v_y = v_{0y} - a_g t,$$

$$y = y_0 + v_{0y}t - \frac{1}{2}a_g t^2, \quad v_y^2 = v_{0y}^2 - 2a_g(y - y_0)$$

assuming upward is positive

- ▶ Recombine horizontal and vertical components of position and velocity using

$$s = \sqrt{x^2 + y^2}, \quad \theta = \arctan\left(\frac{y}{x}\right), \quad v = \sqrt{v_x^2 + v_y^2}, \quad \theta_v = \arctan\left(\frac{v_y}{v_x}\right)$$

- ▶ The maximum height $h \equiv \max(y)$ of a projectile launched with initial vertical velocity v_{0y} is

$$h = \frac{v_{0y}^2}{2a_g}$$

- ▶ The maximum horizontal distance traveled by a projectile is called range $R \equiv \max(x)$. The R of a projectile on a level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is

$$R = \frac{v_0^2}{a_g} \sin(2\theta_0)$$

1.3. Problem solving, graphical analysis ★

- The problem-solving basics for physics problems are
 - Examine the situation to determine which physical principles are involved
 - Make a list of what is given or can be inferred from the problem as stated (identify the knowns)
 - Identify exactly what needs to be determined in the problem (identify the unknowns)
 - Find an equation or set of equations that can help you solve the problem
 - Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units
 - Check the answer to see if it is reasonable: does it make sense?
- Graphs of motion can be used to analyze motion
 - In fact, graphical solutions correspond to solutions that are obtainable via the mathematical methods and equations we learned
 - Recall that y-intercept is the y -value when $x = 0$ or when a line crosses the y -axis. Also recall that slope m is the difference in y -value (rise) divided by the difference in x -value (run) between two points on a straight line. In symbols,

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}$$

- The slope of a graph of displacement x vs time t is velocity v
 - The slope of a graph of velocity v vs time t is acceleration a
 - Average velocity, instantaneous velocity, and acceleration can be obtained by graphical analysis
- The 3 fundamental **physical quantities** are mass, length, and time
 - Their corresponding SI units are kilogram (kg), meter (m), and second (s)
 - Derived units for other physical quantities are products or quotients of the basic units
 - Equations must be dimensionally consistent, that is two terms can be added only when they have the same units
- TODO: rules for the sign for velocity, acceleration, etc.

2. Dynamics

This module explores one and two dimensional dynamics, specifically on

- forces (4.1, 4.5, 4.8)
- Newton's laws of motion (4.2–4.4)
- friction, drag, stress, strain (5.1–5.3)
- uniform circular motion, gravitation (6.1–6.5)
- problem solving (4.6, 4.7) ★
- Kepler's laws (6.6) △.

2.1. Forces and Newton's laws

- Motion is pretty
 - eg. dolphin jumping out of water, pole vaulter, flight of bird, orbit of satellite
- **Dynamics** is the study of motion that considers the causes that affect that motion
 - Usually, causes = forces
 - cf. kinematics is such study that doesn't consider the causes
- **Force** is a push or pull
 - It can be defined in terms of various standards
 - It is a vector having both magnitude and direction

- ▶ The various types of forces are all manifestations of four **fundamental forces** in nature, ie. electromagnetic, gravitational, weak nuclear, and strong nuclear forces
- ▶ A force field surrounds an object creating a force and is the carrier of that force
- **External forces** are any outside forces that act on a body
 - ▶ A **free-body diagram** is a drawing of all external forces acting on a body
 - ▶ Here, the object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot
 - ▶ If vectors act in directions that are not horizontal or vertical, break down the vectors into horizontal and vertical components and draw them on the free body diagram
- Newton's laws of motion are the foundation of dynamics
 - ▶ They provide an example of how physics can be broadly encompassing yet simplistic in describing nature
 - ▶ They are also universal laws because they apply to similar situations on earth and in space
- **First law** states that a body at rest remains at rest, or if in motion remains in motion at a constant velocity, unless acted on by a net external force
 - ▶ Also known as the **law of inertia**
 - ▶ **Inertia** is the tendency of an object to remain at rest or in motion, and is related to an object's mass
 - ▶ **Mass** is the quantity of matter in a substance
- **Second law** states that the acceleration \vec{a} of a system is directly proportional to and in the same direction as the net external force $\sum \vec{F}$ acting on the system, and is inversely proportional to its mass m . In symbols,

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

- ▶ Or equivalently, the more familiar

$$\sum \vec{F} = m\vec{a}$$

- ▶ Recall that acceleration \vec{a} is the rate of change in velocity \vec{v} . This means that a change in the latter's magnitude or direction or both causes the former
 - ▶ An external force is one acting on a system from outside the system, as opposed to **internal forces** which act between components within the system
 - ▶ Here, **system** is defined by the boundaries of an object or collection of objects being observed
 - ▶ **Weight** \vec{w} of an object is the force of gravity acting on an object of mass m , causing the object to experience an acceleration \vec{g} due to gravity. In symbols,
- $$\vec{w} = m\vec{g}$$
- ▶ Recall that if the only force acting on an object is due to gravity, the object is in free fall.
 - ▶ **Friction** is a force that opposes the motion past each other of objects that are touching
 - **Third law** states that whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts
 - ▶ A **thrust** is a reaction force that pushes a body forward in response to a backward force, eg. rockets, planes and cars are pushed forward by thrust
 - Aside from the typical directed force we exert on objects (push or pull), force can manifest in other forms

- ▶ When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface and is called a **normal force** \vec{N} . In symbols,

$$\vec{N} = -\vec{w} = -m\vec{g}$$

- ▶ When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object In symbols,

$$N = w = mg$$

Note: magnitude only, denoted by absolute value, as in $N = |\vec{N}|$, $w = |\vec{w}|$, $g = |\vec{g}|$

- ▶ When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight \vec{w} can be broken down into components that act perpendicular (\vec{w}_\perp) and parallel (\vec{w}_\parallel) to the surface of the plane
- ▶ These components can be calculated using

$$\vec{w}_\parallel = w \sin(\theta) = mg \sin(\theta),$$

$$\vec{w}_\perp = w \cos(\theta) = mg \cos(\theta).$$

- ▶ The pulling force that acts along a stretched flexible connector, such as rope or cable, is called tension force or **tension** \vec{T} . In symbols,

$$\vec{T} = m\vec{g}$$

- ▶ In any inertial frame of reference, Newton's laws have simple forms given above, and all forces are real forces having a physical origin
- ▶ **Inertial frame of reference** is a frame that is not accelerated or rotated
- Some problems will contain various physical quantities, such as forces \vec{F} , acceleration \vec{a} , velocity \vec{v} , and position \vec{x}
 - ▶ You can apply kinematics and dynamics in order to solve these problems of motion
 - ▶ Some problems contain multiple force vectors acting in different directions on an object. Break down all force vectors into horizontal and vertical components and draw a free body diagram
 - ▶ Write the second law in horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction, say along x , then $\sum \vec{F}_x = 0$, otherwise $\sum \vec{F}_x = m\vec{a}_x$
 - ▶ The normal force \vec{N} on an object is not always equal in magnitude to weight \vec{w} of object
 - ▶ If the object is accelerating, $N = |\vec{N}|$ might be less than or greater than $w = |\vec{w}|$
 - ▶ If the object is on an inclined plane, it will always be that $N < w$

2.2. Applying Newton's laws

- **Friction** \vec{f} is a contact force between systems that opposes the motion or attempted motion between them, and depends on both materials involved
 - ▶ Simple friction is proportional to the normal force \vec{N} pushing the systems altogether
 - ▶ Recall that \vec{N} is always perpendicular to the contact surface between systems
 - ▶ The magnitude of static friction f_s between systems stationary relative to one another is

$$f_s \leq \mu_s N$$

where μ_s is the coefficient of static friction which depends on both materials. Note: magnitude only, as in $f = |\vec{f}|$, $N = |\vec{N}|$

- ▶ The magnitude of kinetic friction force f_k between systems moving relative to one another is

$$f_k \leq \mu_k N$$

where μ_k is the coefficient of kinetic friction which also depends on both materials

- In a fluid, the **drag forces** \vec{F}_D acting on an object oppose its motion
 - For large objects (eg. baseball) moving at a velocity \vec{v} in air (which is a fluid, like water),

$$\vec{F}_D = \frac{1}{2} C \rho A \vec{v}^2$$

where C is drag coefficient, A is area of the object facing the fluid, and ρ is the fluid density

- For small objects (eg. bacteria) moving at \vec{v} , drag force is given by Stokes' law as in

$$\vec{F}_D = \vec{F}_S = 6\pi\eta r \vec{v}$$

where r is radius of the object, and η is the fluid viscosity

- Deformation is the change in shape due to an applied force
 - The relationship between deformation and applied force is described by **Hooke's law** as in

$$\vec{F} = k \Delta L$$

where \vec{F} is the applied force, k is a proportionality constant that depends on direction of the force and shape and composition of the object, and ΔL is the amount of deformation (here it's change in length L)

- This relationship can also be written as

$$\Delta L = \frac{1}{Y} \frac{F}{A} L_0$$

where Y is Young's modulus which depends on substance or material, A is cross-sectional area, and L_0 is initial length

- Here, the ratio F/A of force to area is called **stress**, and is measured in N/m^2 . Whereas, the ratio $\Delta L/L_0$ of change in length to initial length is called **strain**, and is a unitless quantity. Then alternatively,

$$\text{stress} = Y \times \text{strain}$$

- The **shear deformation** Δx caused by a force F applied perpendicular to length L_0 and parallel to cross-sectional area A is

$$\Delta x = \frac{1}{S} \frac{F}{A} L_0$$

where S is shear modulus

- In 2-dim, the amount of deformation (here it's change in volume ΔV) caused by a force F per unit area A applied uniformly inward on all surfaces is

$$\Delta V = \frac{1}{B} \frac{F}{A} V_0$$

where B is bulk modulus, and V_0 is initial volume

2.3. Circular motion

- **Uniform circular motion** is motion in a circle at constant speed
 - **Rotation angle** $\Delta\theta$ is the ratio of arc length Δs and radius r of curvature as in

$$\Delta\theta = \frac{\Delta s}{r}$$

where arc length is distance traveled along a circular path

- ▶ Its SI unit is radians (rad), for which $2\pi \text{ rad} = 360 \text{ deg} = 1 \text{ revolution}$ ($1 \text{ rad} = 57.3 \text{ deg}$)
- ▶ **Angular velocity** $\vec{\omega}$ is the rate of change of angle $\vec{\theta}$ as in

$$\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}$$

where a rotation $\Delta\vec{\theta}$ (sort of like a rotational displacement) takes place in a time interval Δt . Its SI unit is rad/s

- ▶ Linear velocity \vec{v} and angular velocity $\vec{\omega}$ are related by

$$\vec{v} = r\vec{\omega} \iff \vec{\omega} = \frac{\vec{v}}{r}$$

- ▶ **Centripetal acceleration** \vec{a}_c is acceleration experienced by an object while in uniform circular motion, and has magnitude

$$a_c = \frac{v^2}{r}, \quad \text{where } a_c = r\omega^2$$

and its SI unit is m/s^2

- ▶ It always points toward the center of rotation, and is perpendicular to linear velocity \vec{v}
- ▶ **Centripetal force** \vec{F}_c is any force causing uniform circular motion, and has magnitude

$$F_c = ma_c \iff F_c = m\frac{v^2}{r} \iff F_c = mr\omega^2$$

- ▶ It is a “center-seeking” force that always points toward the center of rotation, and is perpendicular to linear velocity \vec{v}
- ▶ Rotating and accelerated frames of reference are non-inertial. **Fictitious forces** (eg. Coriolis force) are needed to explain motion in such frames
- **Newton’s universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them
- ▶ The magnitude F of this gravitational force is directly proportional to the product of their masses m and M , and is inversely proportional to the square of the distance between them. In symbols,

$$F = G\frac{mM}{r^2}$$

where $G = 6.674 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ is the gravitational constant

- ▶ This law applies universally
- **Kepler’s laws for planetary motion** are applicable for small mass m orbiting a larger mass M in near-isolation
- ▶ **First law** states that the orbit of each planet about the sun is an ellipse with the sun at one focus
- ▶ **Second law** states that each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times
- ▶ **Third law** states that the ratio of the squares of the periods of any two planets about the sun is equal to the ratio of the cubes of their average distances from the sun. In symbols,

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

where T is period (as in time it takes to orbit once) and r is average radius of the orbit

- ▶ The period and radius of a satellite's orbit about a larger body M are related by

$$T^2 = \frac{4\pi^2}{GM}r^3 \iff \frac{r^3}{T^2} = \frac{G}{4\pi^2}M$$

3. Work, energy, and momentum

This module explores work and energy, specifically on

- Work, energy, W-E theorem, gravitational energy (7.1–7.3)
- Potential energy, energy conservation (7.4–7.6)
- Impulse, momentum conservation (8.1–8.3)
- Elastic, inelastic collisions (8.4–8.6)
- Work, energy, power in humans (7.7–7.9) ★
- Rocket propulsion (8.7) △.

3.1. Work, energy, W-E theorem

- **Work** is the transfer of energy by a force acting on an object as it is displaced
 - ▶ Work W that a force \vec{F} does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,

$$W = Fd \cos \theta$$

- ▶ The SI unit for work and energy is the joule (J), where $1 \text{ J} = 1 \text{ N m} = 1 \text{ kg m}^2/\text{s}^2$
 - ▶ The work done by a force is zero if the displacement is either zero or perpendicular to the force
 - ▶ The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction
- The net work W_{net} is the work done by the net force acting on an object
 - ▶ Work done on an object transfers energy to the object
 - ▶ The translational **kinetic energy** of an object of mass m moving at speed v is

$$\text{KE} = \frac{1}{2}mv^2$$

- ▶ The **work-energy theorem** states that the net work W_{net} on a system changes its kinetic energy, as in

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

- **Power** is the rate at which work is done, or in equation form, for the average power P for work W done over a time t ,

$$P = \frac{W}{t}$$

- ▶ The SI unit for power is the watt (W), where $1 \text{ W} = 1 \text{ J/s}$
 - ▶ The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1 \text{ hp} = 746 \text{ W}$
- Work, energy, and power in humans
 - ▶ The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue

- ▶ The rate at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**, and the corresponding rate when at rest is called the **basal metabolic rate** (BMR)
- ▶ The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next
- ▶ About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate
- ▶ The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food
- ▶ On **world energy use**, the relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing
- ▶ Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources. The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power
- ▶ Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by gross domestic product (GDP) per capita, are matched by higher levels of energy consumption per capita
- ▶ Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes

3.2. Potential energy, energy conservation

- Work done against gravity in lifting an object becomes potential energy of the object-earth system
 - ▶ The **gravitational potential energy** of an object near earth's surface is due to its position in the mass-earth system. Only differences in gravitational potential energy ΔPE_g have physical significance
 - ▶ The change in gravitational potential energy is

$$\Delta PE_g = mgh,$$

with h being the increase in height and g the acceleration due to gravity

- ▶ As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that

$$\Delta KE = -\Delta PE_g$$

- A **conservative force** is one for which work depends only on the starting and ending points of a motion, not on the path taken
 - ▶ We can define **potential energy** PE for any conservative force, just as we defined PE_g for the gravitational force
 - ▶ The potential energy of a spring is

$$PE_s = \frac{1}{2}kx^2,$$

where k is the spring's force constant and x is the displacement from its undeformed position

- ▶ **Mechanical energy** is defined to be $KE + PE$ for a conservative force

- ▶ When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

$$\text{KE} + \text{PE} = \text{constant} \quad \text{or} \quad \text{KE}_i + \text{PE}_i = \text{KE}_f + \text{PE}_f \quad (\text{conservative forces only})$$

where i and f denote initial and final values. This is known as the **conservation of mechanical energy**

- A **nonconservative force** is one for which work depends on the path
 - ▶ Friction is an example of a nonconservative force that changes mechanical energy into thermal energy
 - ▶ Work W_{nc} done by a nonconservative force changes the mechanical energy of a system. In equation form,

$$W_{\text{nc}} = \Delta \text{KE} + \Delta \text{PE},$$

or equivalently,

$$\text{KE}_i + \text{PE}_i + W_{\text{nc}} = \text{KE}_f + \text{PE}_f$$

- ▶ When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws
- The **law of conservation of energy** states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same
 - ▶ When all forms of energy are considered, conservation of energy is written in equation form as

$$\text{KE}_i + \text{PE}_i + W_{\text{nc}} + \text{OE}_i = \text{KE}_f + \text{PE}_f + \text{OE}_f,$$

where OE is all **other forms of energy** besides mechanical energy

- ▶ Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy
- ▶ Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work
- ▶ The **efficiency** eff of a machine or human is defined to be

$$\text{eff} = \frac{W_{\text{out}}}{E_{\text{in}}},$$

where W_{out} is useful work output and E_{in} is the energy consumed

3.3. Impulse, momentum conservation, collisions

- **Linear momentum**, or just **momentum** for brevity, is defined as the product of a system's mass multiplied by its velocity. In symbols,

$$\vec{p} = m\vec{v},$$

where m is the mass of the system and \vec{v} is its velocity

- ▶ The SI unit for momentum is kg m/s
- ▶ **Newton's second law of motion in terms of momentum** states that the net external force equals the change in momentum of a system divided by the time over which it changes. In symbols,

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t},$$

where \vec{F}_{net} is the net external force, $\Delta \vec{p}$ is change in momentum, and Δt is change in time

- **Impulse**, or change in momentum, equals the average net external force multiplied by the time this force acts, as in

$$\Delta \vec{p} = \vec{F}_{\text{net}} \Delta t$$

- Forces are usually not constant over a period of time
- The **conservation of momentum** principle is written as

$$\begin{aligned} \vec{p}_{\text{tot}} &= \text{constant}, & \text{or} \\ \vec{p}_{\text{tot}} &= \vec{p}'_{\text{tot}} & (\text{isolated system}), \end{aligned}$$

where \vec{p}_{tot} is the initial total momentum and \vec{p}'_{tot} is the total momentum some time later

- An **isolated system** is defined to be one for which net external force is zero, as in $\vec{F}_{\text{net}} = 0$
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero
- Conservation of momentum applies only when the net external force is zero
- The conservation of momentum principle is valid when considering systems of particles
- An **elastic collision** is one that conserves internal kinetic energy
 - Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions
- An **inelastic collision** is one in which the internal kinetic energy changes (it's not conserved)
 - A collision in which the objects stick together is sometimes called **perfectly inelastic** because it reduces internal kinetic energy more than does any other type of inelastic collision
 - Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations
- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x -axis parallel to the velocity of the incoming particle
 - Two-dimensional collisions of point masses where mass m_2 is initially at rest conserve momentum along the initial direction of mass m_1 (the x -axis), stated by

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2$$

and along the direction perpendicular to the initial direction (the y -axis) stated by

$$0 = m_1 v'_{1y} + m_2 v'_{2y}$$

- The internal kinetic before and after the collision of two objects that have equal masses is

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_1'^2 + \frac{1}{2} m v_2'^2 + m v_1' v_2' \cos(\theta_1 - \theta_2)$$
- Point masses are structureless particles that cannot spin
- Newton's third law of motion states that to every action, there is an equal and opposite reaction
 - **Acceleration of a rocket** is

$$a = \frac{v_e \Delta m}{m \Delta t} - g.$$

- It depends on three main factors:
- The greater the exhaust velocity of the gases, the greater the acceleration
- The faster the rocket burns its fuel, the greater its acceleration
- The smaller the rocket's mass, the greater the acceleration

4. Torque and rotational motion

This module explores torque and rotational motion, specifically on

- Torque, static equilibrium conditions (9.1–9.3)
- Statics applications (9.4, 9.5)
- Rotational motion (10.1–10.4)
- Angular momentum conservation (10.5, 10.6)
- Forces, torques in muscles and joints (9.6) ★
- Gyroscopic effects (10.7) △.

4.1. Torque, statics

- **Statics** is the study of forces in equilibrium
 - Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration
- The **first condition** necessary to achieve equilibrium is that the net external force on the system must be zero, so that

$$\sum \vec{F} = \vec{0}$$

- The second condition assures those torques are also balanced
 - **Torque** is the rotational equivalent of a force in producing a rotation and is defined to be

$$\tau = rF \sin \theta$$

where τ is torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between \vec{F} and the vector directed from the point where the force acts to the pivot point

- The perpendicular lever arm r_{\perp} is defined to be

$$r_{\perp} = r \sin \theta$$

so that

$$\tau = r_{\perp} F$$

- The perpendicular lever arm r_{\perp} is the shortest distance from the pivot point to the line along which F acts
- The SI unit for torque is newton-meter (N m)
- The **second condition** necessary to achieve equilibrium is that the net external torque on a system must be zero:

$$\sum \vec{\tau} = \vec{0}$$

- By convention, counterclockwise torques are positive, and clockwise torques are negative
- A system is said to be in **stable equilibrium** or **stability** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement
 - A system is in **unstable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium

- ▶ A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position
- Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain
 - ▶ We have discussed the problem-solving strategies specifically useful for statics
 - ▶ Statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in problem-solving strategies, still apply
- **Simple machines** are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we have to apply the force
 - ▶ The ratio of output to input forces for any simple machine is called its mechanical advantage
 - ▶ A few simple machines are the lever, nail puller, wheelbarrow, crank, etc.
- Statics plays an important part in understanding everyday strains in our **muscles and bones**
 - ▶ Many lever systems in the body have a mechanical advantage of significantly less than one, as many of our muscles are attached close to joints
 - ▶ Someone with good posture stands or sits in such a way that the person's center of gravity lies directly above the pivot point in the hips, thereby avoiding back strain and damage to disks

4.2. Rotational motion

- **Uniform circular motion** is the motion with a constant angular velocity

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- ▶ In **non-uniform circular motion**, the velocity changes with time and the rate of change of angular velocity (ie. angular acceleration) is

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

- ▶ **Linear or tangential acceleration** refers to changes in the magnitude of velocity but not its direction, given as

$$a_t = \frac{\Delta v}{\Delta t}$$

For circular motion, note that $v = r\omega$, so that

$$a_t = \frac{\Delta(r\omega)}{\Delta t}$$

The radius r is constant for circular motion, and so $\Delta(r\omega) = r\Delta\omega$ thus

$$a_t = r \frac{\Delta\omega}{\Delta t}$$

By definition, $\alpha = \Delta\omega/\Delta t$ thus

$$a_t = r\alpha, \quad \text{or}$$

$$\alpha = \frac{a_t}{r}$$

- Kinematics is the description of motion

- ▶ The **kinematics of rotational motion** describes the relationships among rotation angle, angular velocity, angular acceleration, and time
- ▶ Starting with the four kinematic equations we developed in the one-dimensional kinematics, we can derive the four rotational kinematic equations (presented together with their translational counterparts) seen in Table 10.2, as in

$$\begin{aligned} \text{rotational} &\implies \text{translational} \\ \theta &= \bar{\omega}t \implies x = \bar{v}t \\ \omega &= \omega_0 + \alpha t \implies v = v_0 + at \\ \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \implies x = v_0 t + \frac{1}{2}at^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\theta \implies v^2 = v_0^2 + 2ax \end{aligned}$$

- ▶ In these equations, the subscript 0 denotes initial values (x_0 and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity \bar{v} are defined as follows:

$$\begin{aligned} \bar{\omega} &= \frac{\omega_0 + \omega}{2}, & \text{and} \\ \bar{v} &= \frac{v_0 + v}{2} \end{aligned}$$

- The farther the force is applied from the pivot, the greater is the angular acceleration; angular acceleration is inversely proportional to mass
- ▶ If we exert a force F on a point mass m that is at a distance r from a pivot point and because the force is perpendicular to r , an acceleration $a = F/m$ is obtained in the direction of F . We can rearrange this equation such that

$$F = ma$$

and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding

$$F = mr\alpha$$

- ▶ Torque is the turning effectiveness of a force. In this case, because F is perpendicular to r , torque is simply

$$\tau = rF$$

If we multiply both sides of the equation above by r , we get torque on the left-hand side. That is,

$$rF = mr^2\alpha, \quad \text{or } \tau = mr^2\alpha$$

- ▶ The **moment of inertia** I of an object is the sum of MR^2 for all the point masses of which it is composed. That is,

$$I = \sum mr^2$$

- ▶ The general relationship among torque, moment of inertia, and angular acceleration is

$$\tau = I\alpha, \quad \text{or } \alpha = \frac{\sum \tau}{I}$$

- The **rotational kinetic energy** KE_{rot} for an object with a moment of inertia I and an angular velocity ω is given by

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2$$

- ▶ Helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades
- ▶ **Work and energy in rotational motion** are completely analogous to work and energy in translational motion
- ▶ The equation for the **work-energy theorem for rotational motion** is

$$\sum W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2$$

- Every rotational phenomenon has a direct translational analog, likewise **angular momentum** L can be defined as

$$L = I\omega$$

- ▶ This equation is an analog to the definition of linear momentum as

$$p = mv$$

The relationship between torque and angular momentum is

$$\sum \tau = \frac{\Delta L}{\Delta t}$$

- ▶ Angular momentum, like energy and linear momentum, is **conserved**. This universally applicable law is another sign of underlying unity in physical laws
- ▶ Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero
- Angular momentum L is analogous to linear momentum and is given by

$$L = I\omega$$

It is changed by torque, following the relationship

$$\sum \tau = \frac{\Delta L}{\Delta t}$$

Angular momentum is conserved if the net torque is zero

$$L = \text{constant}, \quad \text{or}$$

$$L = L' \quad \left(\text{for } \sum \tau = 0\right)$$

This equation is known as the **law of conservation of angular momentum**, which may be conserved in **collisions**

- Torque is perpendicular to the plane formed by r and \vec{F} and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of \vec{F}
 - ▶ The direction of the torque is thus the same as that of the angular momentum it produces
 - ▶ The **gyroscope** precesses around a vertical axis, since the torque is always horizontal and perpendicular to \vec{L}
 - ▶ If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque

$$\vec{L} = \Delta \vec{L},$$

and it rotates about a horizontal axis, falling over just as we would expect

- ▶ Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star

4.3. Circular motion

- **Uniform circular motion** is motion in a circle at constant speed
 - ▶ **Rotation angle** $\Delta\theta$ is the ratio of arc length Δs and radius r of curvature as in

$$\Delta\theta = \frac{\Delta s}{r}$$

where arc length is distance traveled along a circular path

- ▶ Its SI unit is radians (rad), for which $2\pi \text{ rad} = 360 \text{ deg} = 1 \text{ revolution}$ ($1 \text{ rad} = 57.3 \text{ deg}$)
- ▶ **Angular velocity** $\vec{\omega}$ is the rate of change of angle $\vec{\theta}$ as in

$$\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}$$

where a rotation $\Delta\vec{\theta}$ (sort of like a rotational displacement) takes place in a time interval Δt . Its SI unit is rad/s

- ▶ Linear velocity \vec{v} and angular velocity $\vec{\omega}$ are related by

$$\vec{v} = r\vec{\omega} \iff \vec{\omega} = \frac{\vec{v}}{r}$$

- ▶ **Centripetal acceleration** \vec{a}_c is acceleration experienced by an object while in uniform circular motion, and has magnitude

$$a_c = \frac{v^2}{r}, \quad \text{where } a_c = r\omega^2$$

and its SI unit is m/s^2

- ▶ It always points toward the center of rotation, and is perpendicular to linear velocity \vec{v}
- ▶ **Centripetal force** \vec{F}_c is any force causing uniform circular motion, and has magnitude

$$F_c = ma_c \iff F_c = m\frac{v^2}{r} \iff F_c = mr\omega^2$$

- ▶ It is a “center-seeking” force that always points toward the center of rotation, and is perpendicular to linear velocity \vec{v}
- ▶ Rotating and accelerated frames of reference are non-inertial. **Fictitious forces** (eg. Coriolis force) are needed to explain motion in such frames

$$\sum \vec{F} = m\vec{a}$$

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67408 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F_{\perp}/A}{\Delta l/l_0} = \frac{F_{\perp}}{A} \frac{l_0}{\Delta l}$$

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ MPa} = 1 \text{ megapascal}$$

$$W = Fd \quad (\text{or } W = Fd \cos \theta)$$

$$\text{power } P = \frac{\Delta W}{\Delta t}$$

$$K = \frac{1}{2}mv^2$$

$$\sum W = \Delta K = K_2 - K_1$$

$$U_g = mgh$$

$$U_e = \frac{1}{2}kx^2$$

$$\Delta K + \Delta U = 0 \quad (\text{or } K_1 + U_1 = K_2 + U_2)$$

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$$

$$\varepsilon = \frac{W_{\text{out}}}{W_{\text{in}}}$$

$$\text{momentum } \vec{p} = m\vec{v} \quad \sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \vec{J} = \sum \vec{F} \Delta t \vec{J} = \Delta \vec{p} = \vec{p}_2 - \vec{p}_1$$

$$\Delta \vec{P} = 0 \quad (\text{or } \vec{P}_1 = \vec{P}_2) \quad (\text{or } \vec{p}_{1a} + \vec{p}_{1b} + \dots = \vec{p}_{2a} + \vec{p}_{2b} + \dots)$$