Statics and rotations

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2025 W43[1]

[1]Phys 20.01 Mod 4. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Equilibrium ⚖️

Center of mass and gravity 🎯

Quick aside: torque 🔧

Applications in machines and humans 🦾

Quiz time 🕐

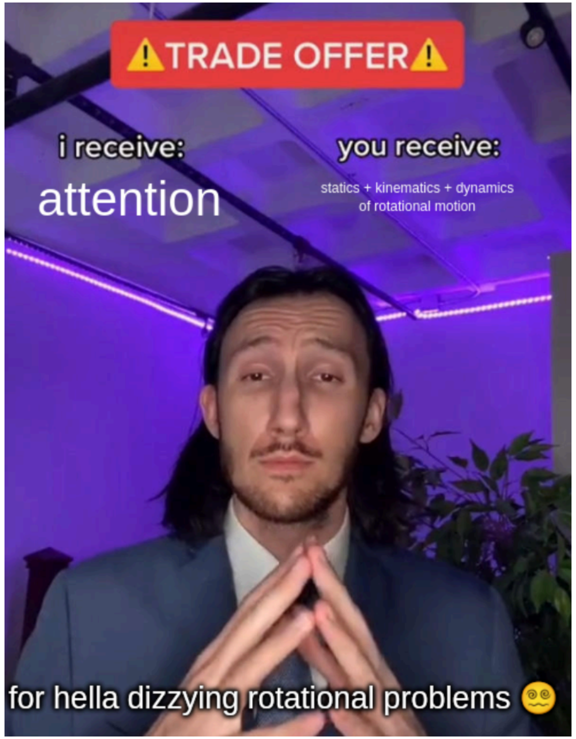
Equilibrium ⚖️

Rotatin’

- What do the motions of ✈️ propeller, blu-ray 💿, and 🎡 have in common?
  - None can be represented adequately as point particle in free-body diagrams
  - Each involves an object that rotates about an axis

Rotatin'

- Rotation 🌀 occurs at all scales, from the motions of electrons in atoms to the motions of entire galaxies
- I offer you general methods for analyzing motion of rotating objects: rotational inertia, torque, angular momentum, *et cetera*



Statics

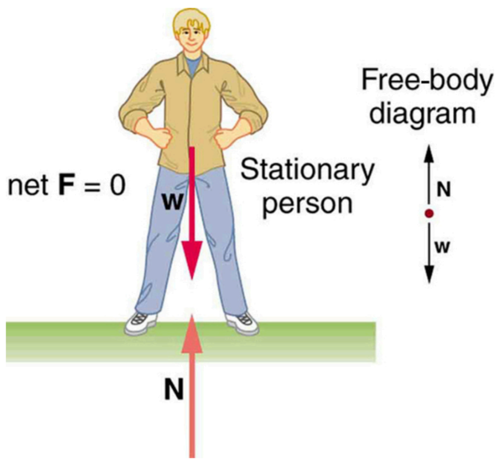
- **Statics** is the study of forces in equilibrium
  - cf. kinematics, dynamics
  - btw, not statistics
- An object is in **equilibrium** if it does not accelerate linearly or rotationally
- Only two conditions must be met to achieve equilibrium

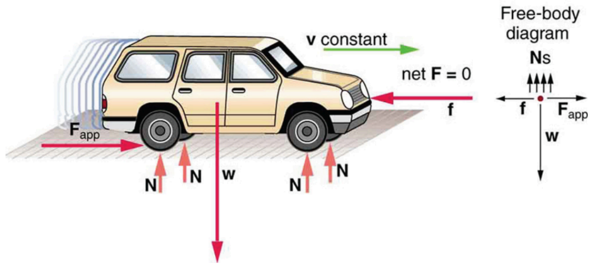
Conditions for equilibrium

- The **first condition for equilibrium** states that for the center of mass of an object at rest to remain at rest, the net external force on the object must be zero:

$$\sum \vec{F} = \vec{0}$$

- Basically, Newton's first law





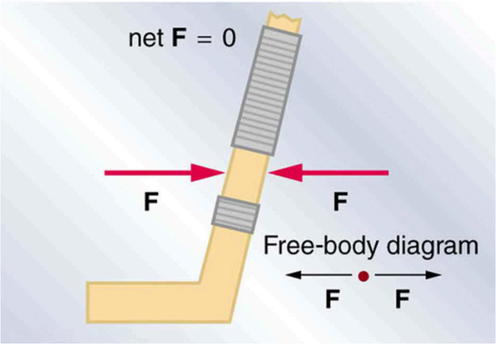
Conditions for equilibrium

- The **second condition** states that for a nonrotating object to remain nonrotating, the net external torque around any point on object must be zero:

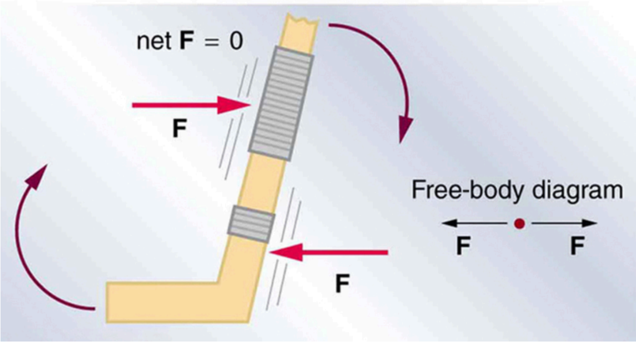
$$\sum \vec{\tau} = \vec{0}$$

- Basically, the object must have no tendency to rotate

Equilibrium: remains stationary

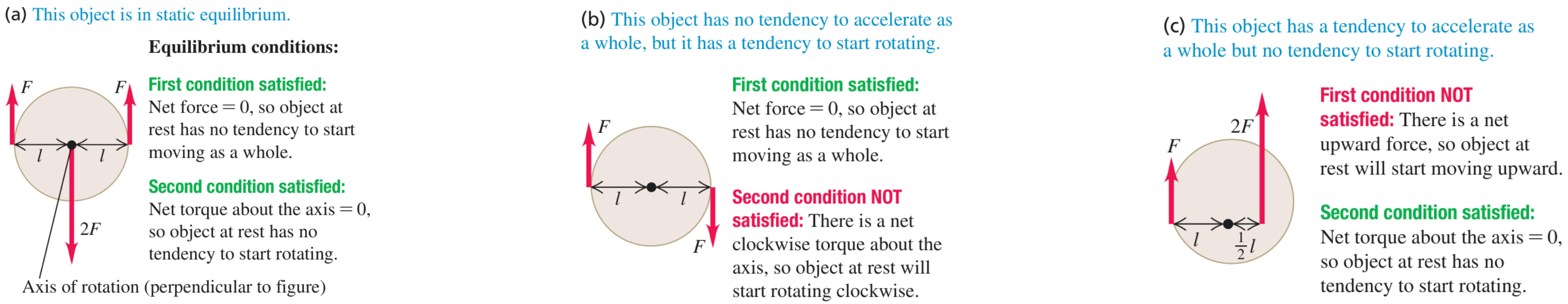


Nonequilibrium: rotation accelerates



Conditions for equilibrium

- If a rigid body is at rest (no translation or rotation), then both first and second conditions apply. Such rigid body or object is said to be in **static equilibrium**

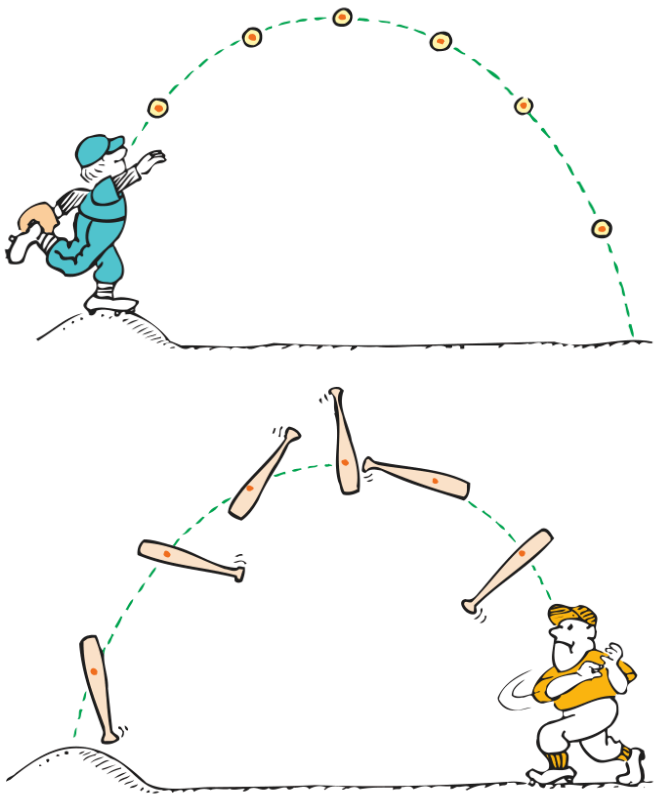


- Otherwise, an object in *uniform translational* motion (constant velocity and no rotation) is still in equilibrium but is not static

Center of mass and gravity 🎯

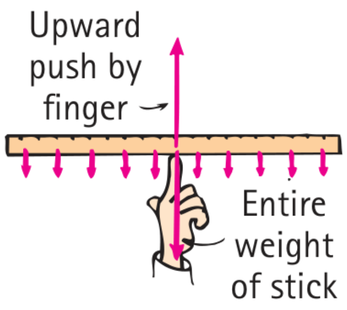
Not so random of a wobble

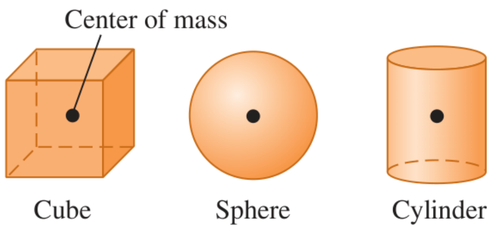
- Toss a baseball ⚾ into the air and it follows a smooth parabolic path
- Toss a baseball bat 🏏 spinning into the air and its path isn't smooth but wobbly
- Seems to wobble all over the place, but it, in fact, wobbles about a very special point



Center of mass and center of gravity

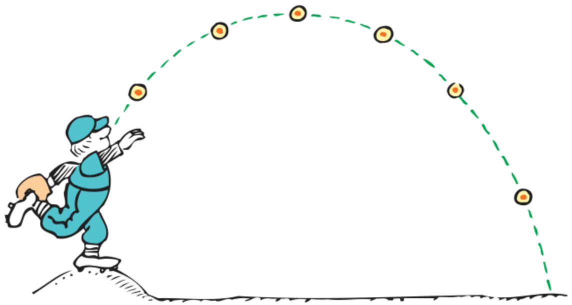
- The weight doesn't act at a single point, it is distributed over the entire object. But we can imagine that the entire force of gravity (weight) is concentrated at a single point
- We refer to this point as **center of gravity** (cg), which is the average position of weight distribution, eg. midpoint of meterstick 📏
- The **center of mass** (cm) is the average position of all mass that makes up the object

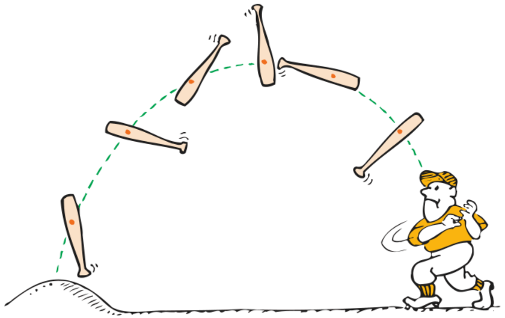




- Since weight and mass are proportional ($w = mg$, then $w \sim m$), both cg and cm mostly refer to the same point of an object

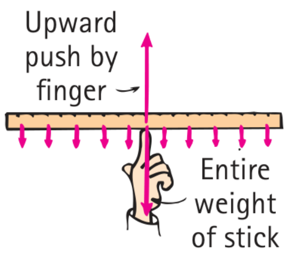
- *Example.* A symmetrical object, such as a ball ⚾, has its cm at its geometrical center. By contrast, an irregularly shaped body, such as a baseball bat 🏏, has more of its mass toward one end. Thus, its cm is toward the thicker end.

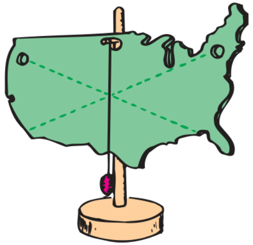


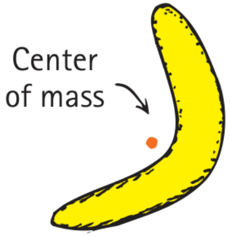


Example. This multiple-flash photograph shows a top view of a wrench 🔧 sliding across a smooth horizontal surface. Note that its cm indicated by white dot follows a straight-line path, while other parts of the wrench wobble around its cm as they move across

Locating the center of gravity/mass

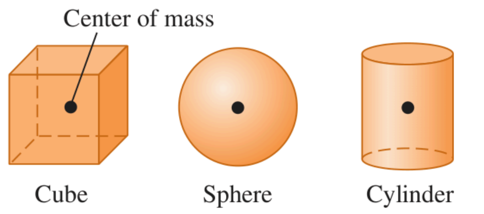




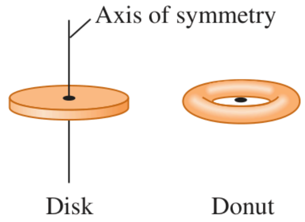


- If you support an object at its cg/cm, you support the entire object. *Balancing* an object is a simple method of locating its cg/cm
- The cg/cm of any freely suspended object lies directly beneath (or at) the point of suspension
- The cg/cm of an object may be a point where no mass exists, eg. a boomerang has its cg/cm outside its physical structure, not within itself

- The cg/cm of homogeneous sphere, cube, or rectangular plate is at its geometric center. The cg/cm of a right circular cylinder or cone is on its axis of symmetry
- For an object with more complex shapes, we can sometimes locate cg/cm by imagining it as being made of symmetrical pieces
  - eg. We could approximate a human body as a collection of solid cylinders, with a sphere for the head (🧍 = 5 🥤 + ⚾)



If a homogeneous object has a geometric center, that is where the center of mass is located.

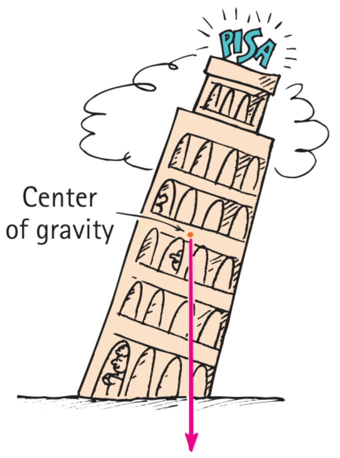


If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

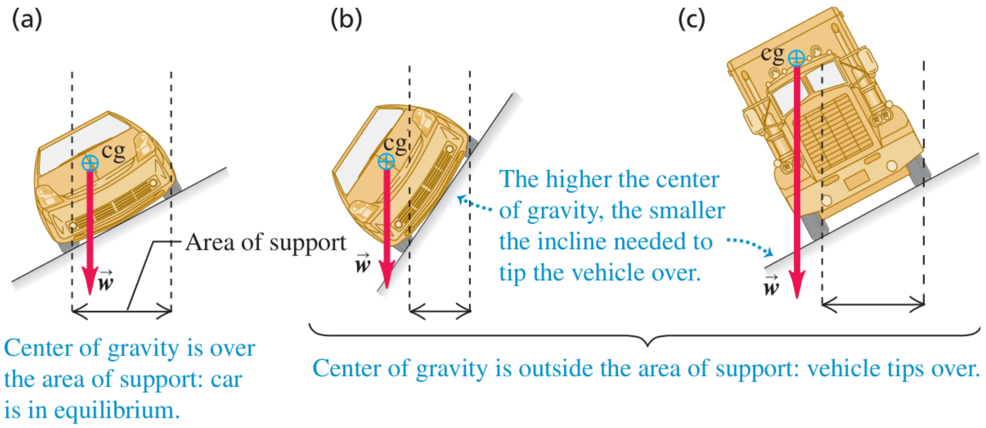
Stability

- The location of cg is important for *stability*. If we draw a line straight down from the cg of an object of any shape and it falls inside the *base* of the object, then the object is in **stable equilibrium** – it will balance. If the line falls outside the base, the object is **unstable**
- The lower the cg and the larger the area of support, the harder it is to overturn an object, eg. four-legged animals 🐈 are naturally stable and need only small feet vs. two-legged animals 🧍 which need relatively large feet

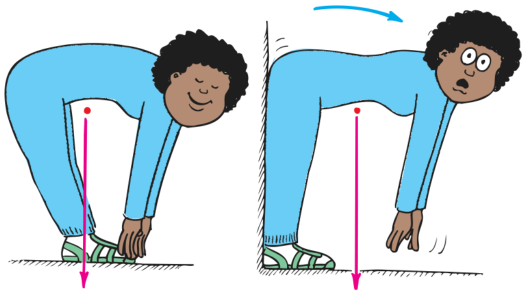
- Simply put, an object whose cg is at or directly above or below the *point of support* is stable and will not rotate
  - eg. The cg of Leaning Tower of Pisa lies above its base of support, so the tower is stable

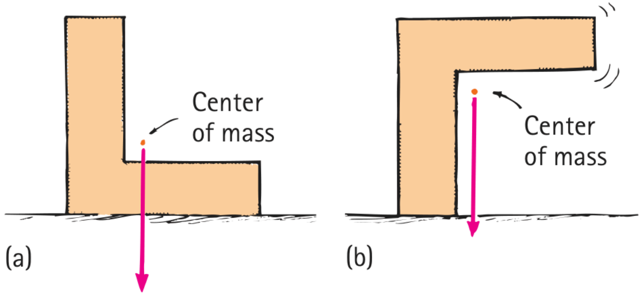


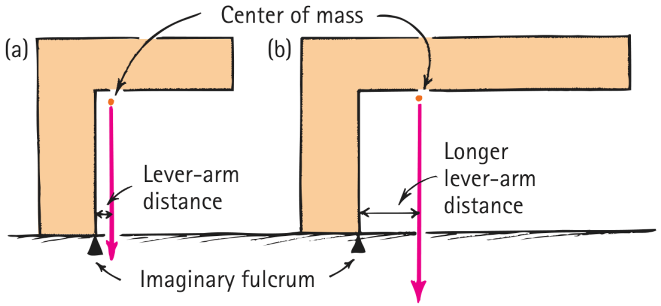
- Similarly, an object supported at several points but whose cg is somewhere within the *area bounded by supports* is also stable and will not tip over

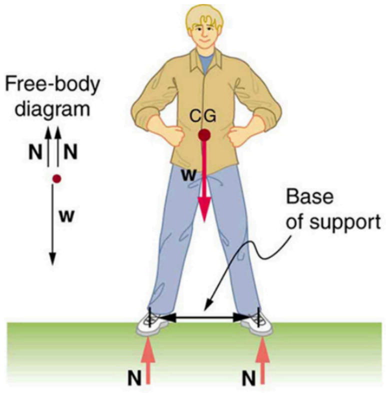


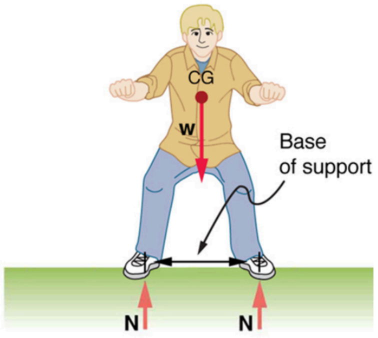
Center of mass and gravity 🎯

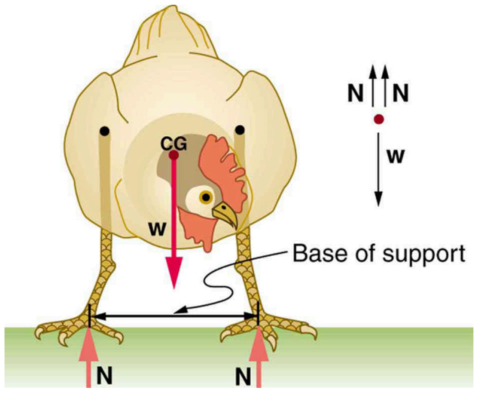






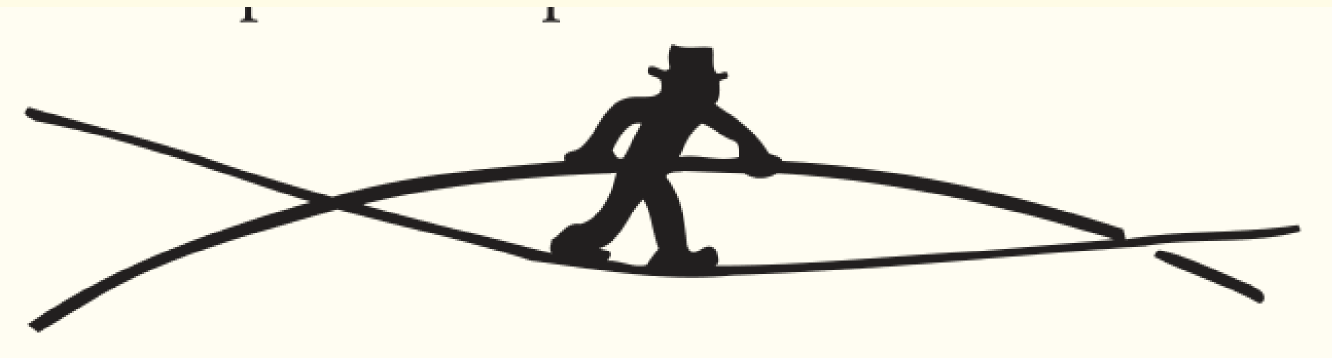






Questions? 🤔

Checkpoint. Why is a long pole more beneficial to a tightrope walker if the pole droops?



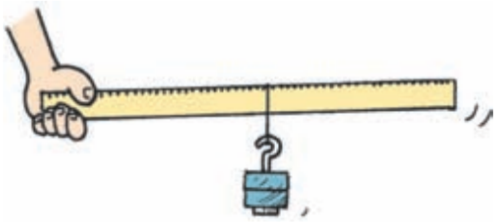
Because it lowers the center of mass.

When the pole droops, its mass is distributed below the tightrope walker's hands and feet. This significantly lower the overall center of mass yielding greater stability

Quick aside: torque 🔧

What is this feeling: it's twistin'

- Hold the end of a meterstick 📏 horizontally with your hand. Dangle a weight 📦 from it near your hand. You can feel the stick *twist*



- Now slide the weight *farther* from your hand and you can feel *greater twist*, even though the weight is the same
- The force acting on your hand is the same. What's different is the *torque*!

Torque

- **Torque** $\vec{\tau}$ (rhymes with *dork*) is the rotational couterpart of force, and is roughly defined as

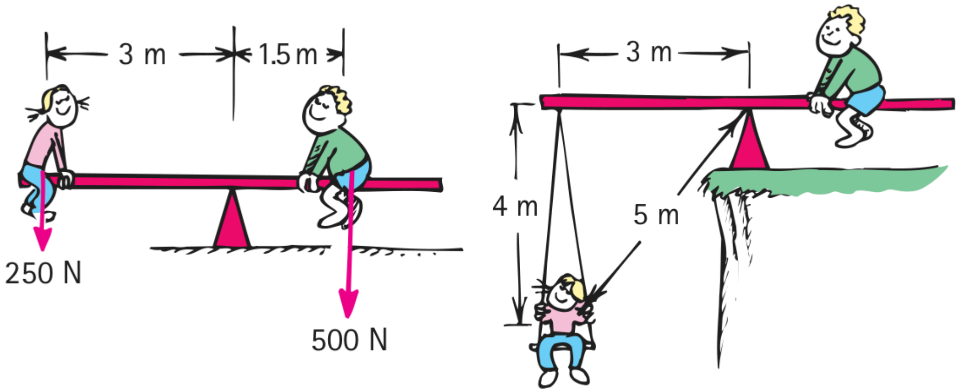
$$\text{torque} = \text{lever arm} \times \text{force}$$

  - It involves distance from the axis of rotation which provides leverage and is called **lever arm**

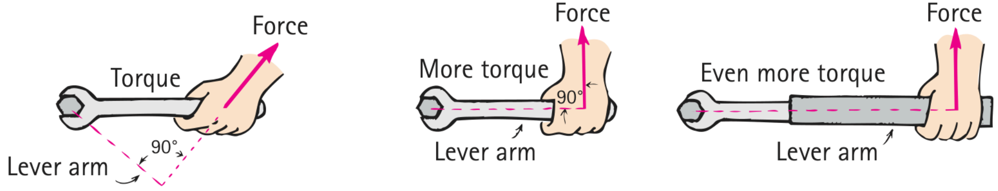
- If *forces* tend to change an object's motion along straight line, then *torques* tend to change its motion via rotations and twists
- No rotation is produced when torques balance out: $\sum \vec{\tau} = \vec{0}$

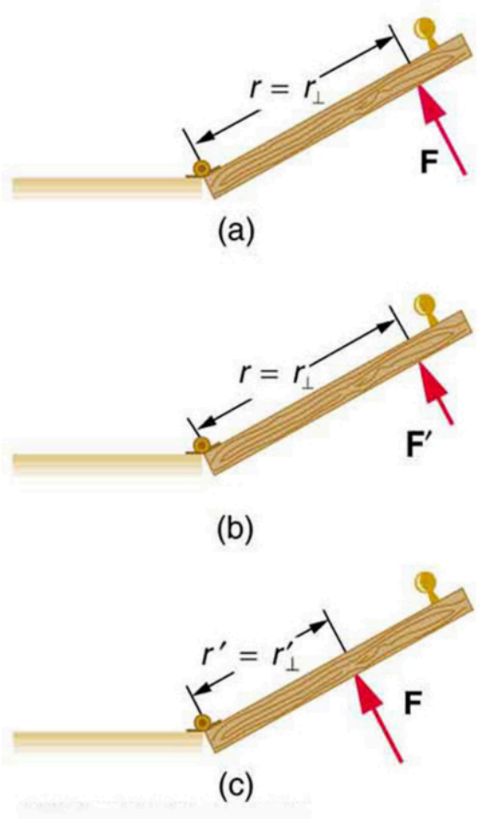
Quick aside: torque 🔧

Example. Weight alone does not produce rotation. Torque (combo of force and lever arm) does. In both cases here, the distance of the girl from the fulcrum (lever arm) is 3 m



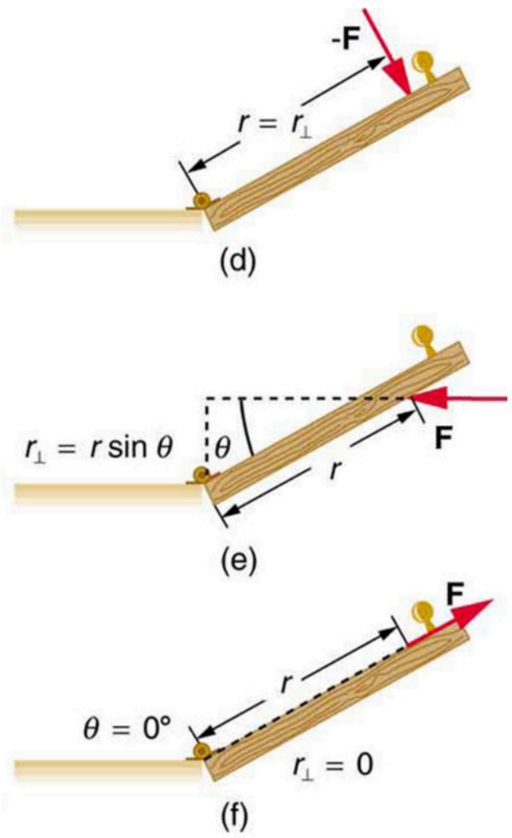
Example. Want more torque? Increase the lever arm, or apply more perpendicular force





Example. Various torques when opening the door

- Positive torque due to force $\vec{F}$ and lever arm $\vec{r}$ causes door to rotate counterclockwise (ccw)
- Positive but smaller torque due to same lever arm but smaller force $\vec{F}' < \vec{F}$ causes door to rotate less ccw
- Positive but smaller torque due to same force but smaller lever arm $\vec{r}' < r$ causes door to rotate less ccw



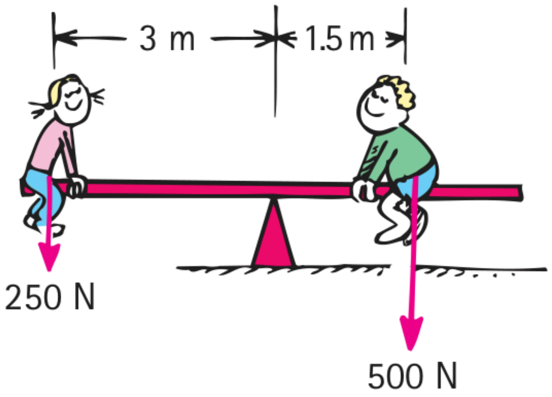
- Negative torque due to same lever arm but force in opposite direction $-\vec{F}$ causes door to rotate clockwise (cw)
- Positive but smaller torque due to same force magnitude but not perpendicular direction relative to the lever arm causes door to rotate less ccw. Notice: $r_\perp = r \sin\theta$
- Zero torque due to same force magnitude but parallel direction relative to the lever arm causes no rotation to the door

Example. If a pipe === effectively extends a wrench handle 🔧 to three times its length, by how much will the torque increase for the same applied force?

- Three times more leverage for the same force produces three times more torque τ, as in

$$3\tau = (3 \times \text{lever arm}) \times \text{force}$$

- Caution: this method of increasing torque sometimes results in shearing off the bolt!



Example. Consider a balanced seesaw. Suppose the girl (weighs 250 N) on the left suddenly gains 50 N after being handed a bag of apples. Where should she sit in order to maintain the balanced seesaw, assuming the heavier boy (weighs 500 N) does not move? 🧺🍏

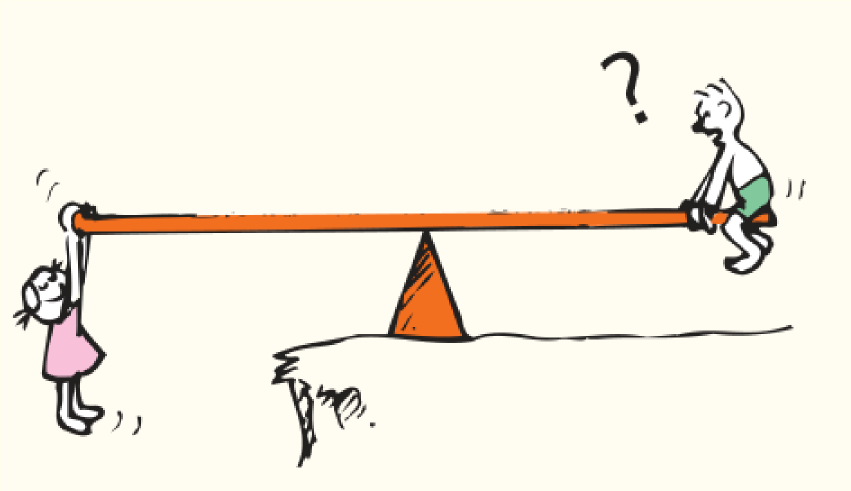
- The boy's torque is $\tau_{👦} = 500\ \text{N} \times 1.5\ \text{m} = 750\ \text{N m}$. The girl should match this torque, that is $\tau_{👧} = \tau_{👦} = 750\ \text{N m}$

$$\tau_{👧} = (250 + 50)\ \text{N} \times (\text{new lever arm})_{👧} = 750\ \text{N m}$$

$$\implies (\text{new lever arm})_{👧} = (750\ \text{N m})/(300\ \text{N}) = 2.5\ \text{m}$$

Questions? 🤔

Checkpoint. Is the net torque changed when a partner on a seesaw stands or hangs from her end instead of sitting? Does the weight or the lever arm change?



No, the net torque is generally not changed when she stands or hangs provided they she maintains the same horizontal position relative to the pivot (lever arm).

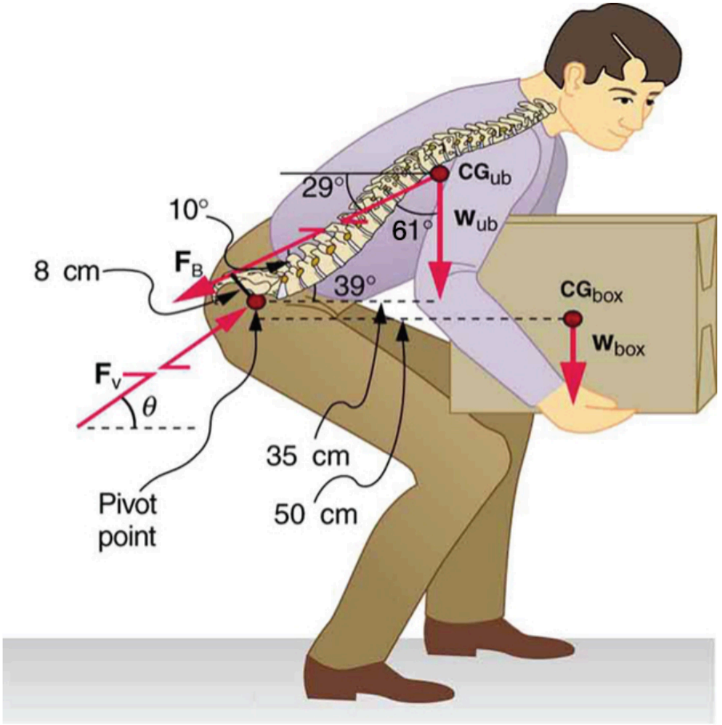
Neither the weight (force) nor the effective lever arm changes, so torque is the same.

Recall: roughly, torque = force × lever arm

Applications in machines and humans 🦾

Applications

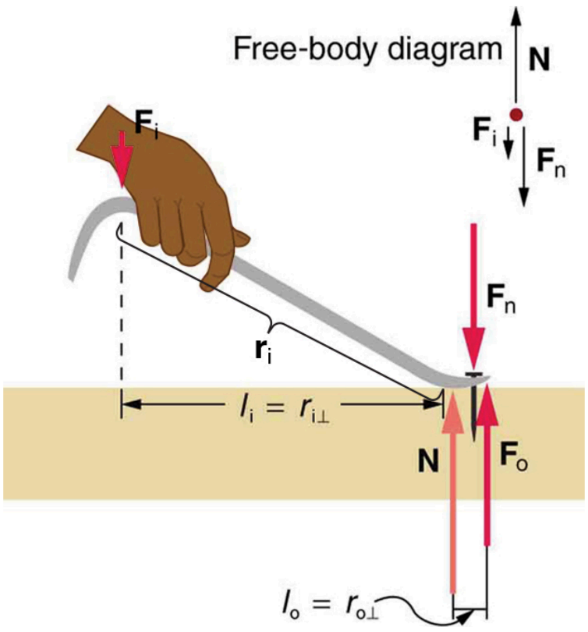
- Statics and rotations can be applied to many situations, eg. raising a drawbridge 🌁, bad posture, back pain 🧓
- Basically, they're a special case of Newton's laws. The strategies we learned for Newton's laws still apply



In machines

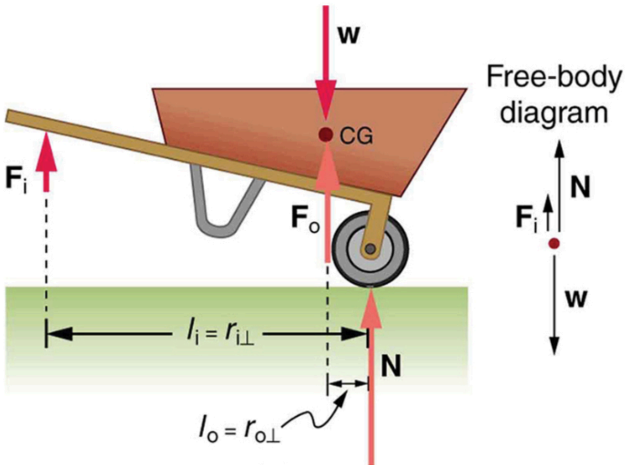
- Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance (lever arm) through which we have to apply the force
  - They reduce the input force that is needed to perform a job
- The ratio of output to input forces for any simple machine is called its **mechanical advantage** A_m as in

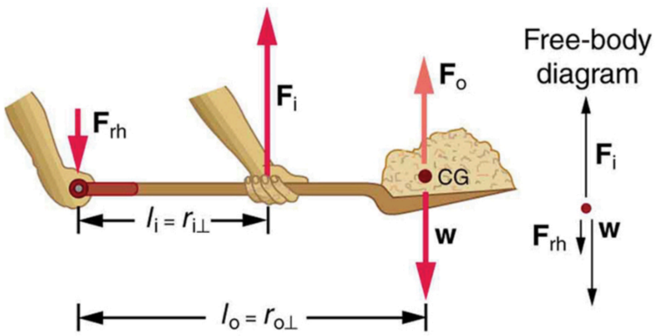
$$A_m = \frac{F_{\text{out}}}{F_{\text{in}}}$$



Example. A *nail puller* is a *lever* with a large A_m. It amplifies the force F_i from the hand into force F_o that will pull the nail up. The perpendicular lever arms of the input and output forces are l_i and l_o. Torques are involved in levers since there is rotation about a pivot point (also called *fulcrum*)

- The torques due to F_i and F_o must be equal to each other if the nail is not moving to satisfy second condition for equilibrium ($\sum \tau = 0$). Thus its mechanical advantage is $A_m = F_o/F_i = l_i/l_o$



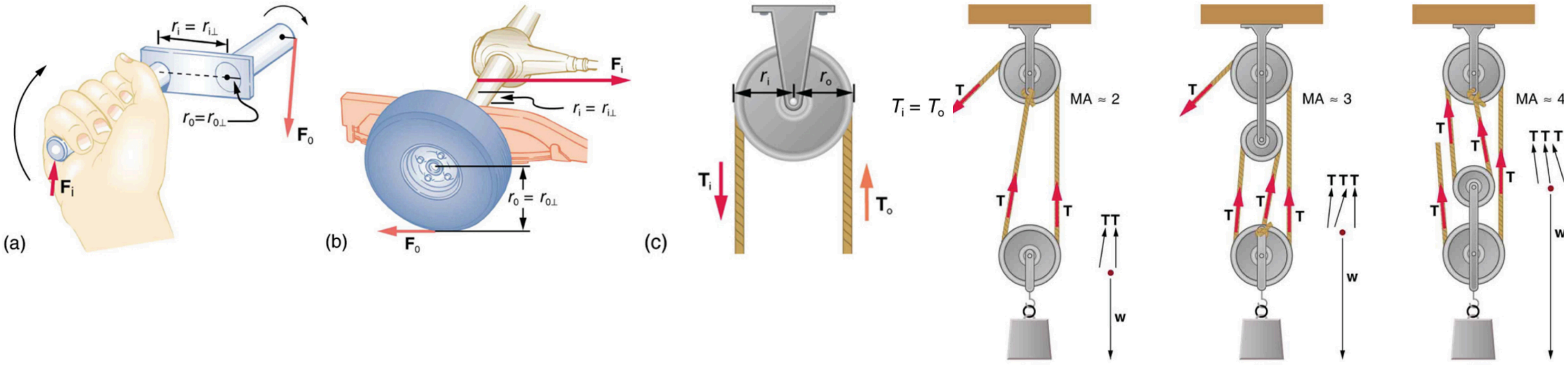


Example. The *wheelbarrow* and *shovel*, also levers, differ from nail puller because both the input and output forces are on the same side of the pivot

- 🛒: output force or load is between pivot (wheel axle) and input force
- 🪏: input force is between pivot (end of handle) and load, but input lever arm is shorter than output's. So its A_m is smaller and less than one

Example. (a) A *crank* is a lever that can be rotated about its pivot. They are designed to have a large A_m

- (b) A simplified automobile axle drives a wheel, which has a much larger diameter than the axle. It has $A_m < 1$



- (c) An ordinary pulley is used to lift a heavy load. The pulley changes the direction of the force exerted by the cord without changing its magnitude. It has $A_m = 1$
- Combinations of pulleys multiply force and increases mechanical advantage A_m (denoted as MA in figures above)

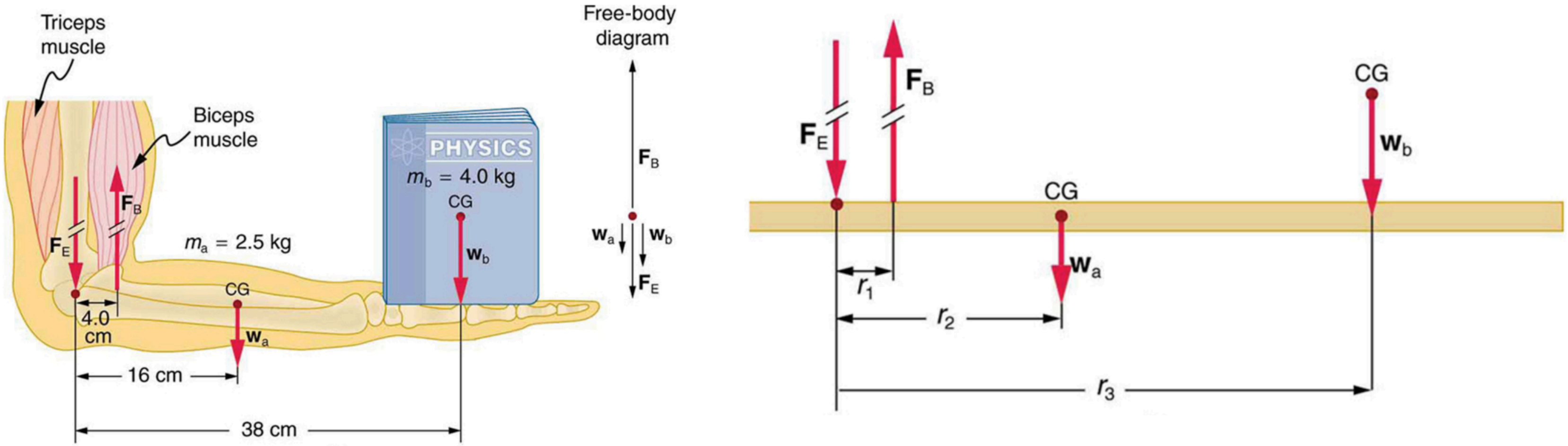
Example. Inclined planes are another type of simple machines. Pushing a cart up an incline is easier than lifting the same cart straight up to top using a ladder because the applied force is less

- However, they do not involve torques and rotations, only the ideas of work and gravitational potential energy (if you recall)

In humans

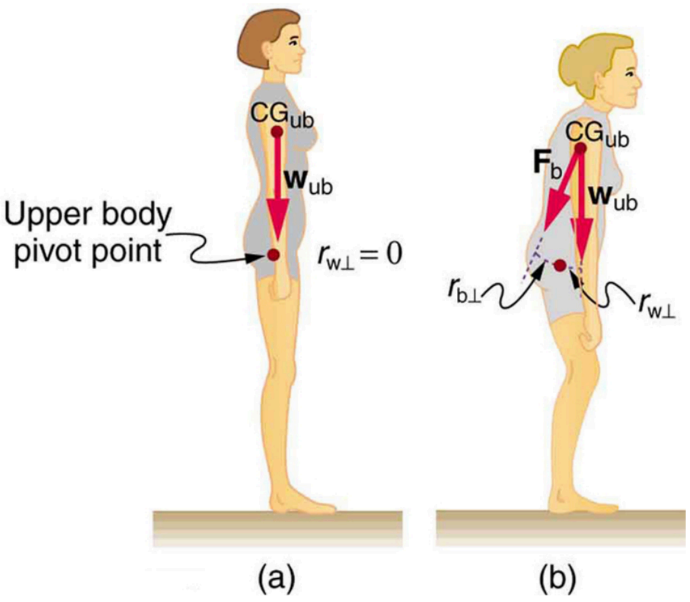
- Statics and rotations play an important part in understanding everyday strains in our *muscles and bones*
- Most skeletal muscles exert much larger forces within the body than the limbs apply to outside world. And so, many of these lever systems in our body have mechanical advantage of much much less than one ($A_m \ll 1$), as many of our muscles are attached close to joints
- Viewing them as simple machines, input force exerted by muscles (eg. bicep-tricep pair) is much greater than output force

Example. The biceps exert a force to support the weight of the forearm and the book. The triceps are assumed to be relaxed. An equivalent system with the pivot at the elbow joint is shown



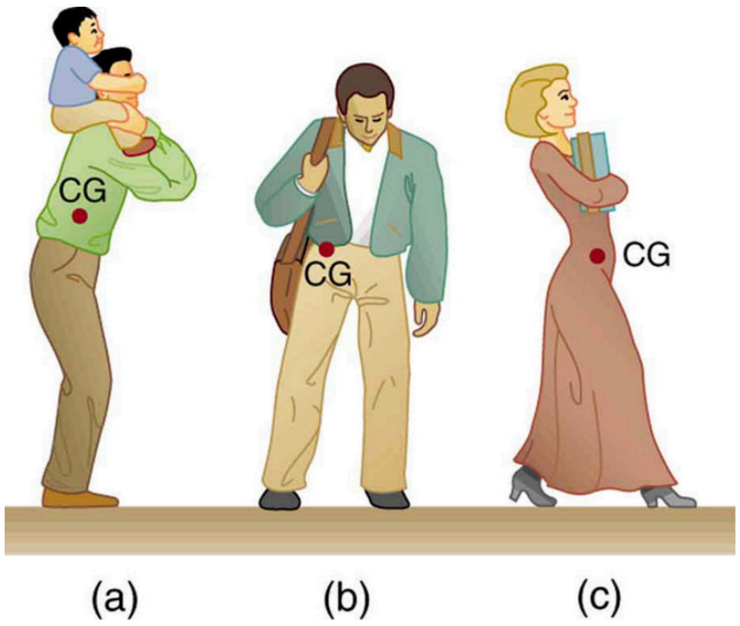
- Muscles exert far greater forces than we might think. Here, biceps exerts a force that is 7.38 times the weight supported

Example. (a) Good posture places the upper body's cg over pivots in the hips, eliminating the need for muscles to balance the body. (b) Poor posture requires exertion by back muscles to counteract clockwise torque produced around pivot by upper body's weight



- Back muscles have small effective perpendicular lever arm $r_{b\perp}$ and must therefore exert a large force $\vec{F}_b$
- Tip: Stand or sit in such a way that your cg lies directly above pivot point of your hips, thereby avoiding strain and damage to your spinal discs

Example. People adjust their stance to maintain balance.

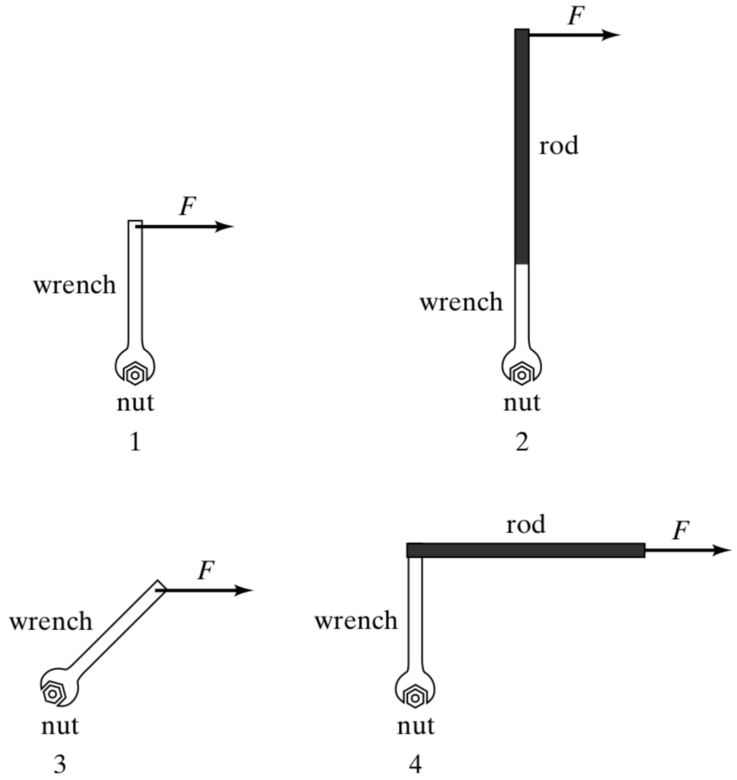


- A father carrying his son piggyback leans forward to position their overall cg above base of support at his feet
- A student carrying a shoulder bag leans to the side to keep the overall cg over his feet
- Another student carrying a load of books in her arms leans backward for the same reason

Quiz time 🕐

Loosening a rusty nut 🔧

You are using a wrench and trying to loosen a rusty nut. Which of the setups shown is most effective in loosening the nut? List in order of descending efficiency the following arrangements.



Bonus! Take a train ride 🚂

Pick a friend to go with you to somewhere accessible via a train ride. This is an experiment to perform while standing in a bus or a train. Record a video of the surroundings (and yourself and friend) as you do as follows then answer/discuss the prompt questions. Your face does not have to be in the frame of the video and you can do the discussion not in public if you prefer. Also pls tell us where you're headed a la vlog, only if you're okay with that

Note: For your safety and those around you, make sure you are holding onto something while you carry out this activity

- Stand facing sideways 🚂 🧍
  - How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates?
- Now stand facing forward 🚂 🚶
  - How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates?
- Is it easier and safer to stand facing sideways rather than forward? Why or why not? 🤔