Momentum, impulse, and collisions

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2025 W42[1]

[1] Phys 20.01 Mod 3. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Previously

Momentum and impulse 🧙

Conservation of momentum ⚛️

Collisions 💥

Quiz time 🕐

Rocket propulsion 🚀

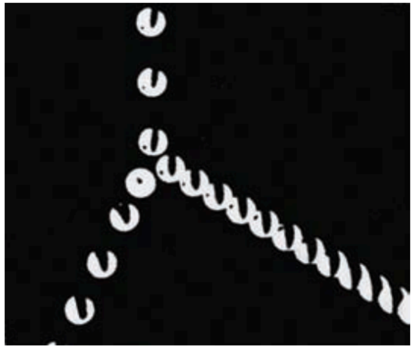
Previously

Potential energy, (non-)conservative forces, conservation of energy, work-energy-power in humans

Momentum and impulse 🧙

Ein moment, bitte

- Many questions involving forces can't be answered by directly applying Newton's second law or work-energy
- eg. in playing pool, how do you decide how to aim the cue ball in order to knock the eight ball into the pocket?



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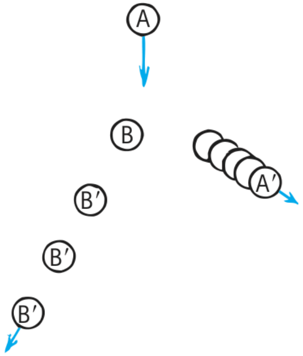
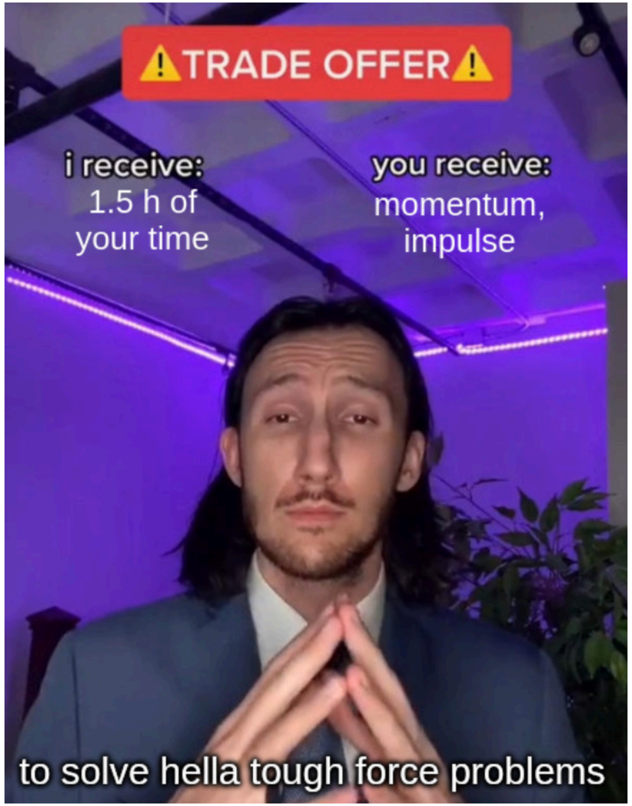


Ein moment, bitte

- eg. when a truck collides head-on with a compact car, what determines which way the wreckage moves after the collision?
- I offer you new approach: two new concepts, *momentum* and *impulse*, and a new law, *conservation of momentum*



Ein moment, bitte

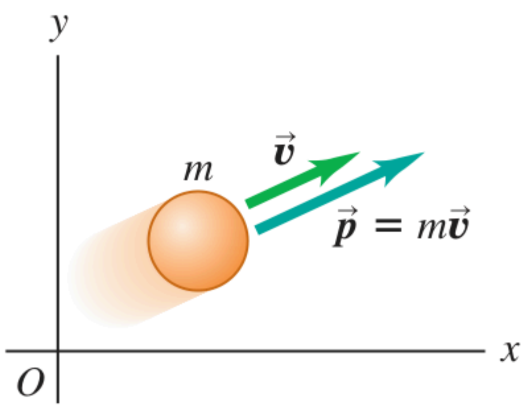
- We all know that a heavy truck is harder to stop than a small car moving at the same speed. We state this fact by saying that the truck has more momentum than the car
- And if two cars have the same mass, the faster one is harder to stop than the slower one . So we also say that the faster-moving car has more momentum than the slower one
- By momentum we mean *inertia in motion*

Momentum

- **Momentum**, or **linear momentum** to be precise, is a vector quantity equal to the product of the system's mass m and velocity $\vec{v}$, as in

$$\vec{p} = m\vec{v}$$

- The greater the mass m and speed v, the greater the momentum magnitude



Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

Momentum

- Its SI unit is kg m/s
- Newton's second law of motion, succintly defined as $\sum \vec{F} = m\vec{a}$, can actually be expressed in terms of momentum
  - Observe that both definitions are multiplied by mass m
  - It then states that the net external force $\vec{F}_{\text{net}} = \sum \vec{F}$ on a system is equal to the rate of change in momentum of the system's momentum, as in

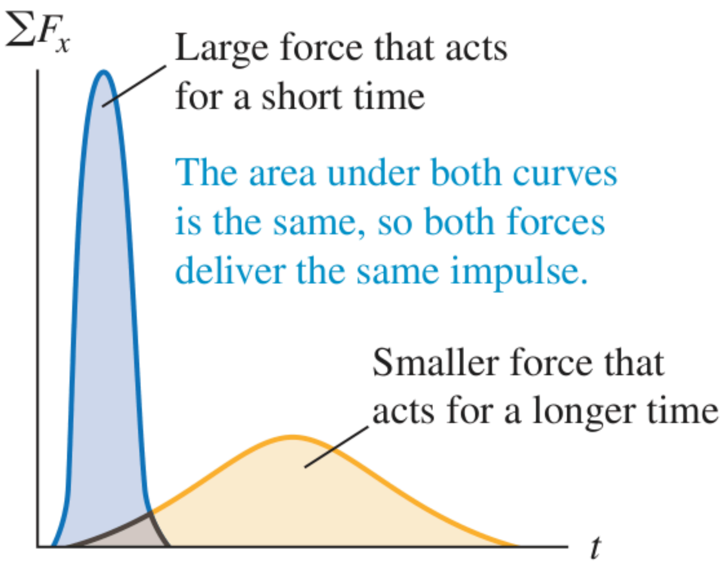
$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \qquad \left(\sum \vec{F} = \frac{m \Delta \vec{v}}{\Delta t} = m \frac{\Delta \vec{v}}{\Delta t} = m\vec{a} \right)$$

Impulse

- **Impulse** of a constant net external force is the product of net external force and time interval over which this force acts, as in

$$\vec{J} = \sum \vec{F} \Delta t$$

- Note that forces are usually not constant over a period of time
- Its SI unit is newton-second (N s), or kg m/s, same for momentum



Example. We see that a huge ship ($\uparrow\uparrow m$) moving at a low speed ($\downarrow v$) can have a large momentum ($\uparrow p$). As is a small bullet ($\downarrow m$) moving at a very high speed ($\uparrow\uparrow v$)

Example. The boulder, unfortunately, has more momentum than the runner

Example. When you push with same force for twice the time, you impart twice the impulse: $2 \cdot \vec{J} = \sum \vec{F} \cdot (2 \cdot \Delta t)$

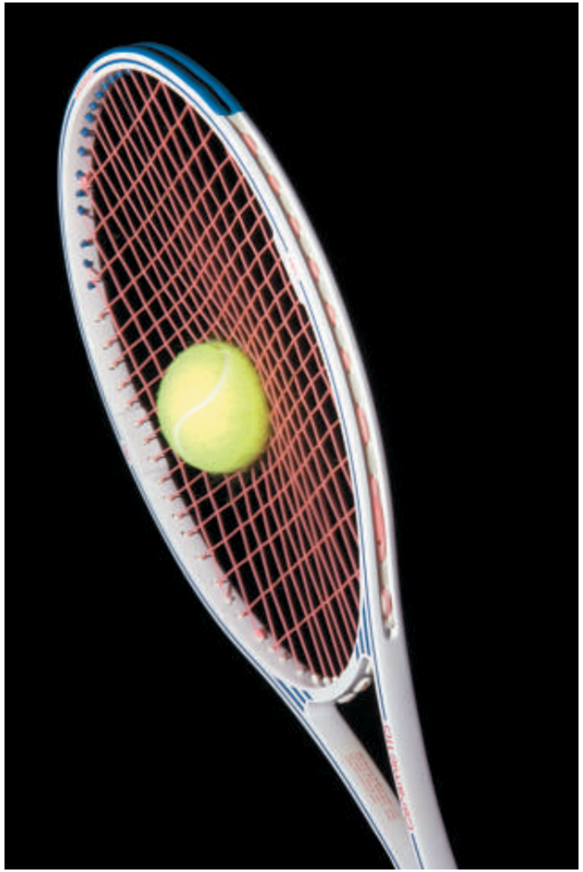
Example. Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of hard-thrown 0.410-kg football with speed 25.0 m/s

- $\vec{p}_{🏃} = (110 \text{ kg})(8.00 \text{ m/s}) = 880 \text{ kg m/s}$
- $\vec{p}_{⚽} = (0.410 \text{ kg})(25.0 \text{ m/s}) = 10.3 \text{ kg m/s}$
- The ratio of the player's momentum to that of the ball is

$$\frac{\vec{p}_{🏃}}{\vec{p}_{⚽}} = \frac{880 \text{ kg m/s}}{10.3 \text{ kg m/s}} = 85.9$$

- Although ⚽ has greater velocity, 🏃 has a much greater mass. Thus momentum of 🏃 is much greater than that of ⚽

Example. Typically, a tennis ball is in contact with the racket for approximately 0.01 s. The ball flattens noticeably due to the tremendous force exerted by the racket



If we think about this in terms of impulse, we can imagine that the force must be very huge to deform the ball given the very small time: $\left[\uparrow \vec{J}\right] = \left[\uparrow\uparrow \sum \vec{F}\right][\downarrow \Delta t]$

Quick aside: woodpecker impulse

- The pileated woodpecker has been known to strike its beak against a tree up to 20/s and up to 12000/dy
- Impact force can be as much as 1200 $\times$ weight of its head
- Because impact lasts such a short time, the impulse $\vec{J}$ is relatively small

Quick aside: woodpecker impulse

- Recall impulse is (force during impact) $\times$ (duration of impact)
- Also the woodpecker has a thick skull of spongy bone as well as shock-absorbing cartilage at the base of the lower jaw, and so avoids injury 🐦 💥

Questions? 🤔 💥

Impulse changes momentum

- Recall the Newton's second law of motion in terms of momentum derived above. Using this in definition of impulse,

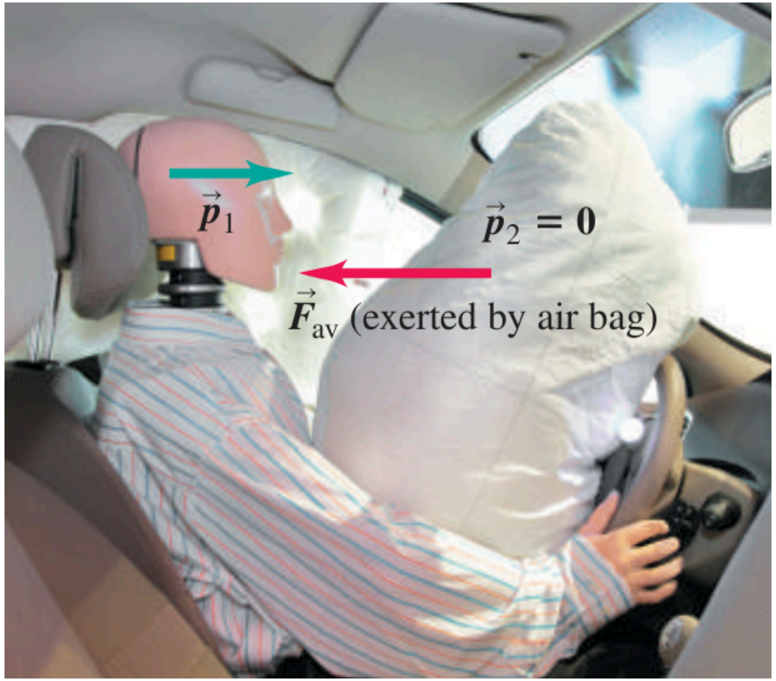
$$\vec{J} = \sum \vec{F} \cdot \Delta t = \frac{\Delta \vec{p}}{\Delta t} \cdot \Delta t = \Delta \vec{p}, \qquad \text{or}$$

$$\vec{J} = \Delta \vec{p} = \vec{p}_2 - \vec{p}_1$$

- This is also known as the **impulse-momentum theorem**, which states that the impulse of the net external force on a system during a time interval equals the change in momentum of that system during that interval

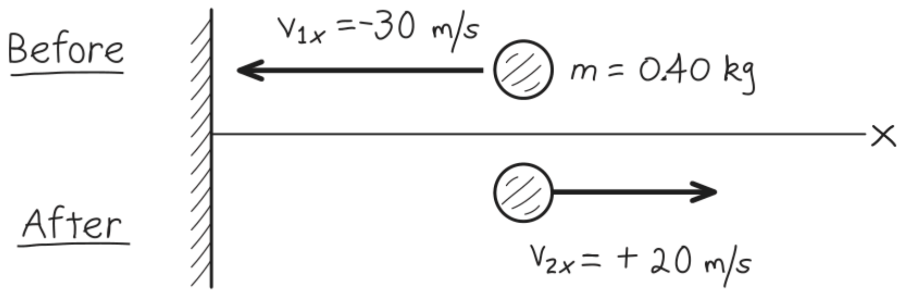
- The greater the impulse exerted on something, the greater will be its change in momentum
- Sometimes the impulse can be considered to be the cause of a change of momentum, sometimes it's the other way around 🔁
- But we only consider ordinary cases wherein impulse relates to increasing momentum ⬆️ , decreasing momentum over a long time ⬇️ 🕔 , and decreasing momentum over a short time ⬇️ 🕐

Example. The impulse-momentum theorem explains how air bags reduce the chance of injury by minimizing the force on an occupant of a car



- $\vec{J}$-$\vec{p}$ theorem: $\vec{J} = \vec{p}_2 - \vec{p}_1 = \vec{F}_{ave}\Delta t$
- Impulse is same no matter how the driver is brought to rest, so $\vec{p}_2 = \vec{0}$
- Compared to striking the steering wheel, striking the air bag brings the driver to rest over a longer time interval Δt
- Hence with an air bag, average force $\vec{F}_{ave}$ on the driver is less

Example. You throw a ball with a mass of 0.40 kg against a brick wall. It is moving horizontally to left at 30 m/s when it hits the wall. It rebounds horizontally to right at 20 m/s. Find the impulse of the net external force on the ball during its collision with wall



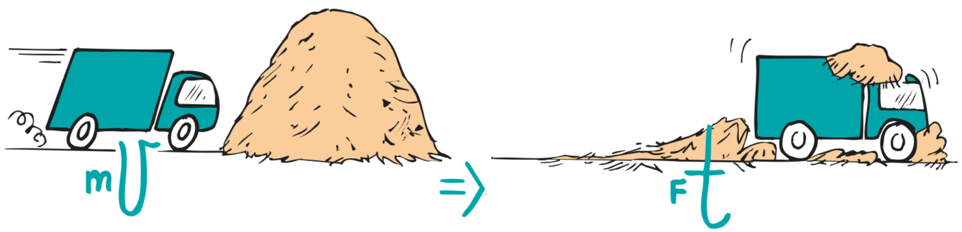
- $p_{1x} = mv_{1x} = (0.40\ \text{kg})(-30\ \text{m/s}) = -12\ \text{kg m/s}$
- $p_{2x} = mv_{2x} = (0.40\ \text{kg})(+20\ \text{m/s}) = +8.0\ \text{kg m/s}$
- $J_x = p_{2x} - p_{1x} = 8.0\ \text{kg m/s} - (-12\ \text{kg m/s}) = 20\ \text{N/s}$

Case 1: increasing $\vec{p}$

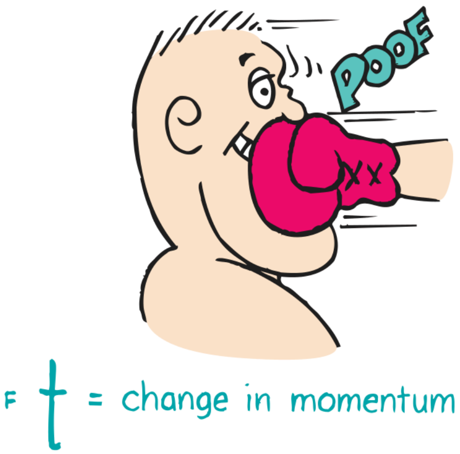
- To increase the momentum of an object, it makes sense to apply the greatest force possible for as long as possible
- eg. A golfer 🏌 teeing off and a baseball player 🏏 trying for a home run do both of these things when they swing as hard as possible ($\uparrow \vec{F}$) and follow through with their swings, as latter extends the time of contact ($\uparrow \Delta t$)
- Btw, forces of impact usually varies throughout duration of impact, eg. 🏌 exerts no force until contact, then increases as ball distorts, then diminishes as ball speeds up and un-deforms

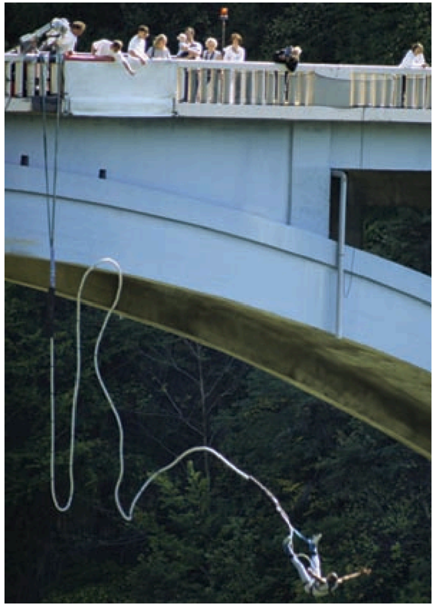
Case 2: decreasing $\vec{p}$ over a long time

- Quick question 🤔 : if you were in a car that was out of control, would you rather hit a concrete wall 🧱 or a haystack 🥦 ?
  - No physics needed, common sense tells haystack because soft
  - Tho physics-wise, haystack because you extend the time Δt during which your $\vec{p}$ is brought to zero ($\vec{J}$-$\vec{p}$ theorem)
  - eg. if time interval is extended $100\times$, force is reduced to $\frac{1}{100}$th
- Whenever we wish force to be small, we extend time of contact



Example. If the change in momentum occurs over a long time, the hitting force is small



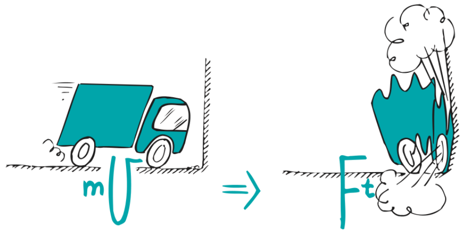


Example. When the boxer moves away (rides with the punch), he extends the time and diminishes the force

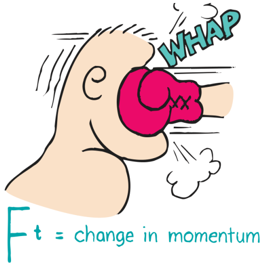
Example. A large momentum change over a long time requires a safely small average force

Case 3: decreasing $\vec{p}$ over a short time

- For an object brought to rest, the impulse is the same no matter how it is stopped. But if time is short, the force will be large



Example. If the change in momentum occurs over a short time, the hitting force is large



Example. If boxer moves into gloved fist, time is reduced and he must withstand a greater force

Example. Cassy imparts a large impulse to the bricks in a short time and produces a huge force

Questions? 😵 🥊

Checkpoint. Do moving objects have impulse? Do moving objects have momentum?

No, impulse is not something an object has, unlike momentum. It is what an object can provide or what it can experience when it interacts with some other object. An object cannot possess impulse just as it cannot possess force 💥

Yes, but like velocity in a relative sense (ref frame). The momentum possessed by a moving object with respect to a stationary point on earth may be quite different from the momentum it possesses with respect to another moving object 💥

Brain break! 🧠 💤

Viewin’ veritasium 🤓

Watch one of Derek Muller’s classics:

- youtu.be/dQw4w9WgXcQ
- youtu.be/XGxIE1hr0w4
- youtu.be/Hx9TwM4Pmhc
- youtu.be/8leAAwMIigI

Conservation of momentum ⚛️

Momentum conservation

- The **principle of conservation of momentum** states that if the vector sum of external forces on a system is zero, then the total momentum $\vec{P}$ of system of objects $a, b, \ldots$ is constant, as in

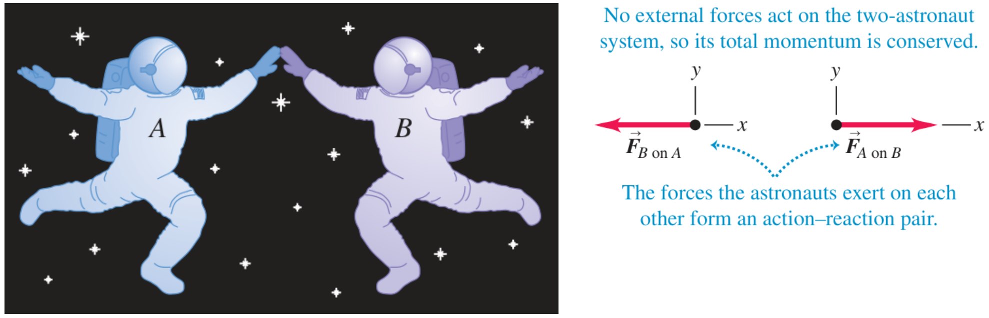
$$\Delta\vec{P} = \vec{0} \qquad \text{or}$$

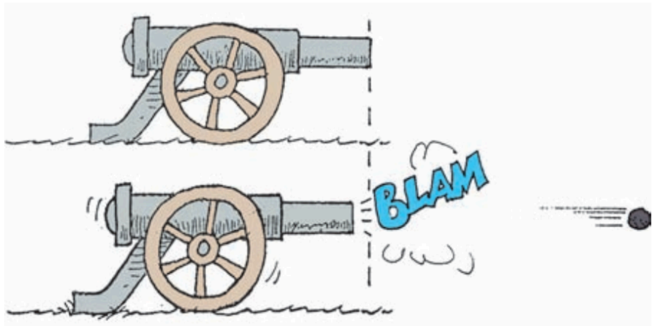
$$\vec{P}_1 = \vec{P}_2 \qquad \text{or}$$

$$\vec{p}_{1a} + \vec{p}_{1b} + \cdots = \vec{p}_{2a} + \vec{p}_{2b} + \cdots$$

where $\vec{P} = \vec{p}_{\text{net}} = \sum\vec{p} = \vec{p}_a + \vec{p}_b + \cdots = m_a\vec{v}_a + m_b\vec{v}_b + \cdots$

- Conservation of momentum applies only when the net external force is zero, and is valid when considering systems of objects
- An **isolated system** is such system wherein net external force is zero, as in $\sum \vec{F} = 0$
- eg. during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero





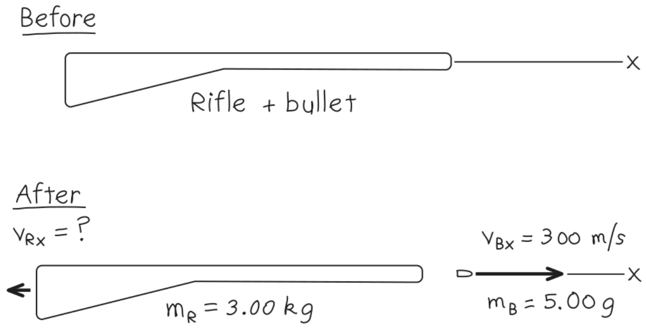
Example. The momentum before firing is zero. After firing, the net momentum is still zero because momentum of cannon is equal and opposite to momentum of cannonball

8-ball system cue-ball system cue-ball + 8-ball system

Example. A cue ball hits an 8 ball head-on. Consider this event in three systems. Only the third system has conserved momentum

Example. A marksman holds a rifle of mass $m_r = 3.00$ kg loosely, so it can recoil freely. He fires a bullet of mass $m_b = 5.00$ g horizontally with a velocity relative to ground of $v_{bx} = 300$ m/s. What is the recoil velocity v_{rx} of the rifle?

- Rifle and bullet start at rest so conservation of total x-momentum gives



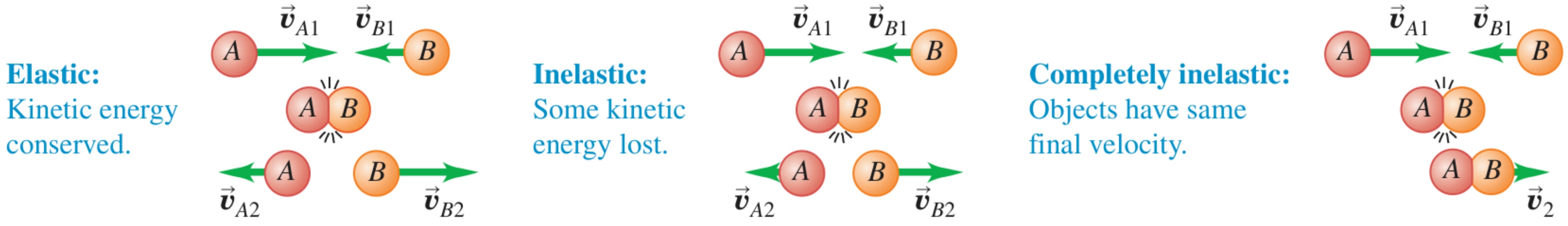
$$\Delta P_x = 0 \Longrightarrow 0 + 0 = m_b v_{2bx} + m_r v_{2rx}$$

$$v_{rx} = v_{2rx} = -\frac{m_b}{m_r} v_{2bx} = -\left(\frac{0.005 \text{ kg}}{3 \text{ kg}}\right)(300 \text{ m/s}) = -0.5 \text{ m/s}$$

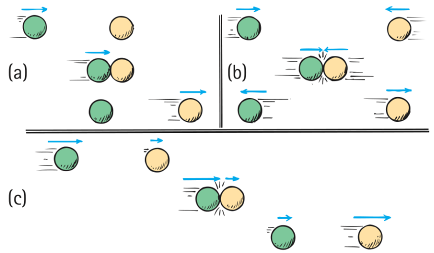
Collisions 💥

Collision

- Momentum is conserved in collisions, ie. net momentum of a system of colliding objects is unchanged before, during, and after the collision
  - This is because the forces that act during collision are internal forces, as in forces acting and reacting within the system itself
  - There is only redistribution or sharing of whatever momentum exists before the collision
- In any collision, net momentum before collision equals net momentum after collision (basically, conservation of momentum)

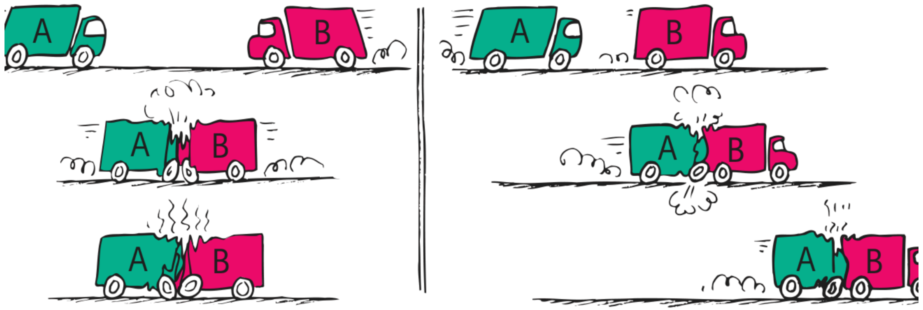


- **Elastic collision** is one that conserves internal kinetic energy
  - Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions



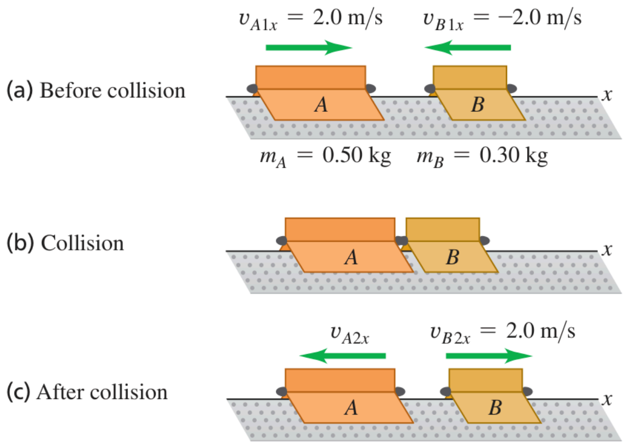
Example. Elastic collisions of equally massive balls. In each case, momentum is transferred from one ball to the other

- **Inelastic collision** is one in which the internal kinetic energy changes or is not conserved
  - When colliding objects stick together or when same final velocity is attained, we have a **perfectly inelastic**. Here, internal kinetic energy reduces more than does any other type of inelastic collision



Example. The net momentum of trucks before and after the collision is the same

Example. Two gliders with different masses move toward each other on a frictionless air track. After they collide, glider B has a final velocity of +2.0 m/s. What is the final velocity of glider A?



- Before the collision,

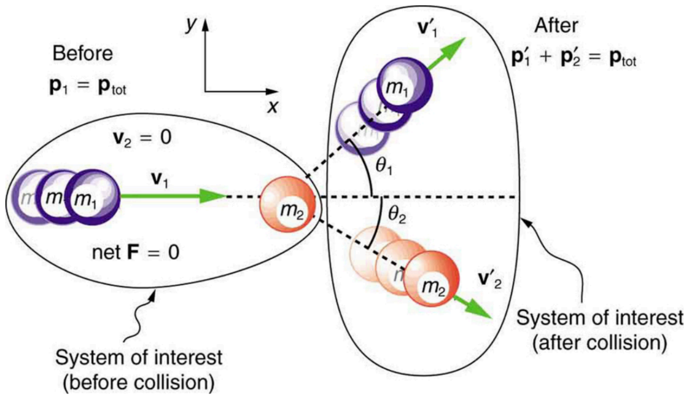
  $P_{1x} = m_a v_{a1x} + m_b v_{b1x} = (0.5 \text{ kg})(2 \text{ m/s})$
  $+ (0.3 \text{ kg})(-2 \text{ m/s}) = 0.4 \text{ kg m/s}$

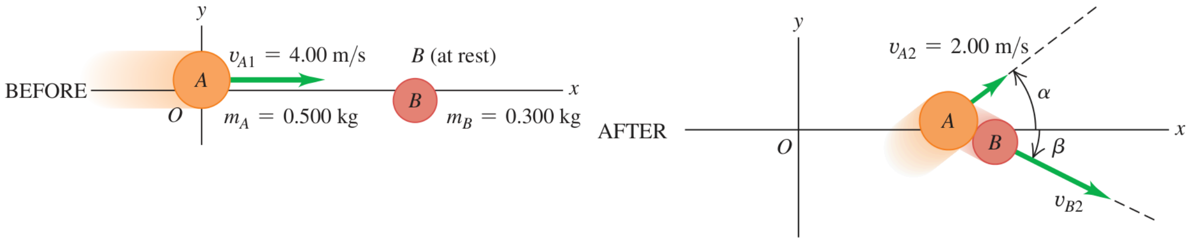
- After the collision and with conservation of momentum, we isolate final velocity

$$v_{a2x} = \frac{P_{1x} - m_b v_{b2x}}{m_a} = \frac{(0.4 \text{ kg m/s}) - (0.3 \text{ kg})(2 \text{ m/s})}{0.5 \text{ kg}} = -0.4 \text{ m/s}$$

- For two-dimensional collisions, choose a convenient coordinate system and break the motion into components along perpendicular axes. Then use your right triangles, cos, and sin:

$$m_a v_{a1x} = m_a v_{a2x} \cos\theta_{a2} + m_b v_{b2x} \cos\theta_{b2},$$
$$0 = m_a v_{a2y} \sin\theta_{a2} + m_b v_{b2y} \sin\theta_{b2}$$





Quick aside: inelastic car

Cars are designed so that collisions are inelastic. The structure of the car absorbs as much of the energy of collision as possible. This absorbed energy cannot be recovered since it goes into a permanent deformation of the car 💥 🚗

Questions? 😵 💥

Quiz time 🕐

Drop that egg 🐣

Design and construct a case 🧺 to hold an egg 🥚 that can and will be dropped from the third floor of Faura without breaking. The design cannot include means to increase air resistance, eg. parachute 🪂, so all cases should strike ground with about the same speed. Hint: $\min\left(\vec{J}\right)$

We'll test your cases next sesh. You'll be judged on design (50%) and protection (50%). The top 3 least broken eggs will get +2 on quizzes 💥

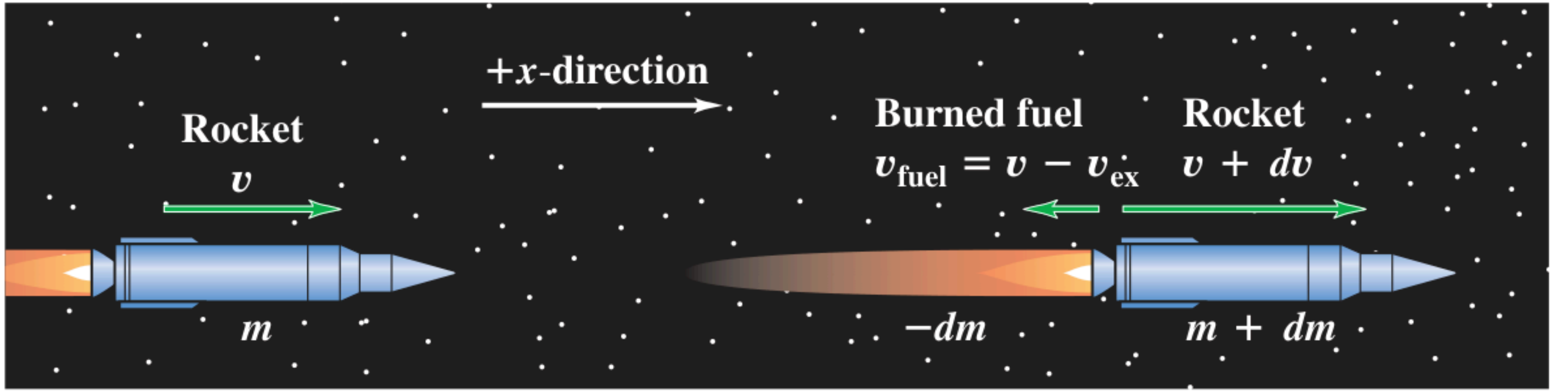
Rocket propulsion 🚀

Rocket, eh?

- Newton's third law of motion states that to every action, there is an equal and opposite reaction
- Acceleration $\vec{a}_{🚀}$ of a rocket of mass m ejecting gas of mass Δm:

$$\vec{a}_{🚀} = \frac{\vec{v}_e}{m}\frac{\Delta m}{\Delta t} - g.$$

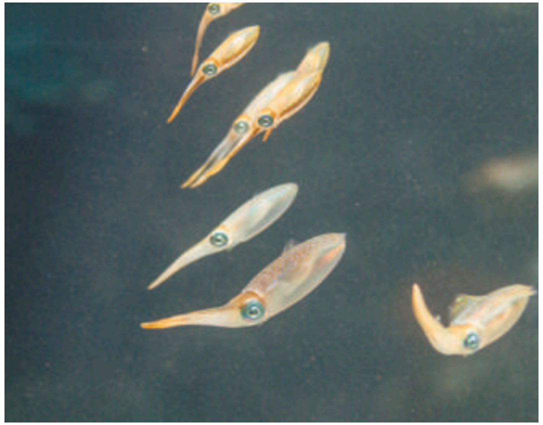
  where $\vec{v}_e$ is exhaust velocity, Δt is how long gas was ejected
- The greater exhaust velocity of gases, the greater acceleration
- The faster rocket burns its fuel, the greater its acceleration
- The smaller the rocket's mass, the greater the acceleration



At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{fuel} = v - v_{ex}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)

Quick aside: jet propulsion in squids



- Both a jet engine and a squid use variations in their mass to provide propulsion
- They increase their mass by taking in fluid (air for a jet engine, water for a squid) at low speed, then decrease their mass by ejecting that fluid at high speed
- The Caribbean reef squid (Sepioteuthis sepioidea) can use jet propulsion to vault to a height of 2 m above the water and fly a total distance of 10 m, about 50 times its body length! 🦐 💨