Work and energy, continued

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2025 W42[1]

[1] Phys 20.01 Mod 3. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Previously

Potential energy 🥁

(Non-)conservative forces 🌀

Conservation of energy ⚛️

Quiz time 🕐

Work, energy, and power in humans 🧑

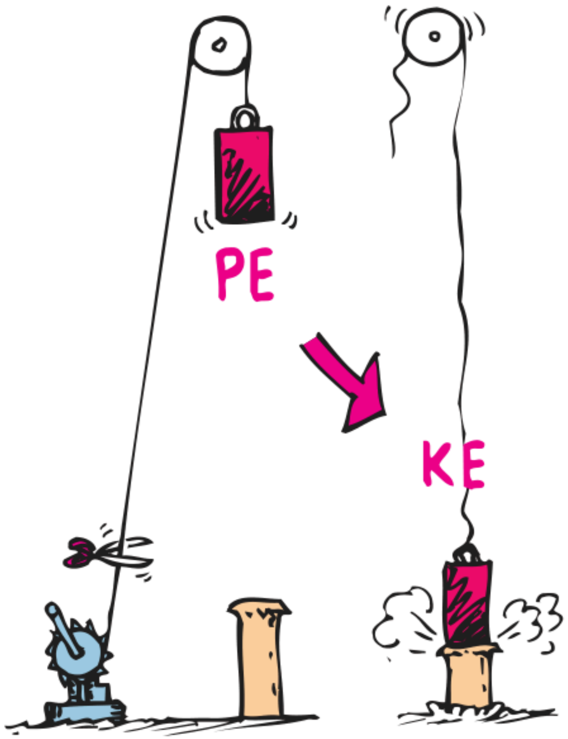
Previously

Work, energy, work-energy theorem, power

Potential energy 🥁

Mechanical energy

- Recall the two common forms of mechanical energy: energy due to position of something (potential) and due to motion of something (kinetic)
- Mechanical energy can be in the form of potential energy, kinetic energy, or the sum of the two



Mechanical energy

- As discussed, the kinetic energy K of an object of mass m moving at speed v is

$$K = \frac{1}{2}mv^2$$

- Here, downhill "fall" of 🎢 results in roaring speed in the dip, and this K sends it back up to the next summit

Potential energy

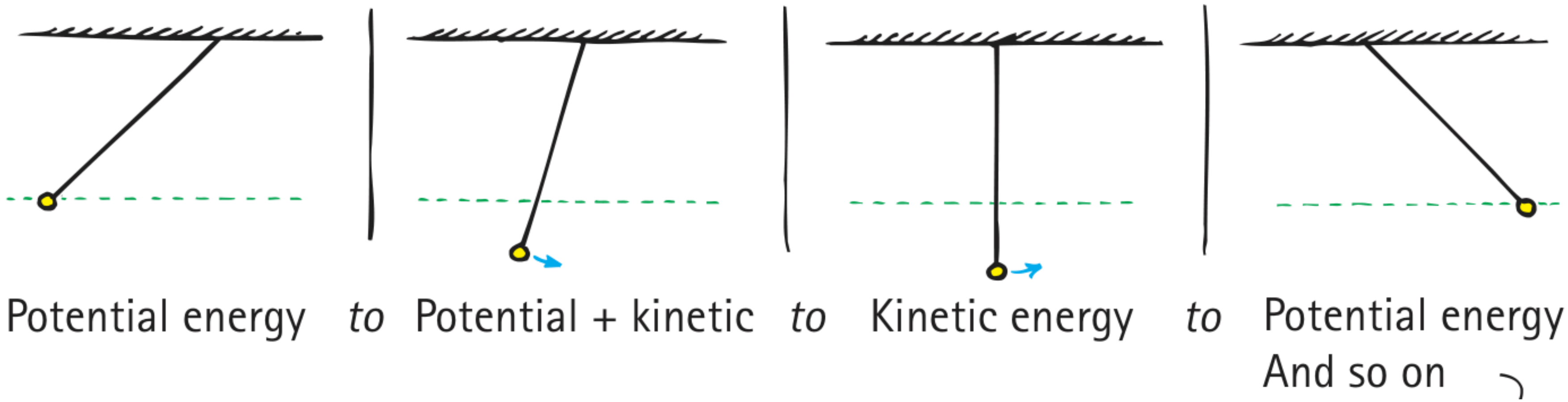
- An object may store energy by virtue of its position
- The energy that's stored and held in readiness is known as **potential energy** U, because in the stored state it has the potential for doing work
  - eg. Stretched/compressed spring 🪱 has potential to do work
  - eg. When 🏹 is drawn, energy is stored in the 🏹 . The bow can do work on the arrow ↙
  - eg. Stretched rubber band has U due to relative positions of its parts. If part of a slingshot, it is capable of doing work

Potential energy

Example. The potential energy U of Tenny's drawn bow 🏹 equals the work ($W = Fd$) that she did in drawing the arrow ↙ back into position. When the ↙ is released, most of the U of the drawn 🏹 will become the kinetic energy K of the ↙

Potential energy

Example. Energy transitions in a pendulum. Potential energy U is relative to the lowest point of the pendulum, when it is vertical

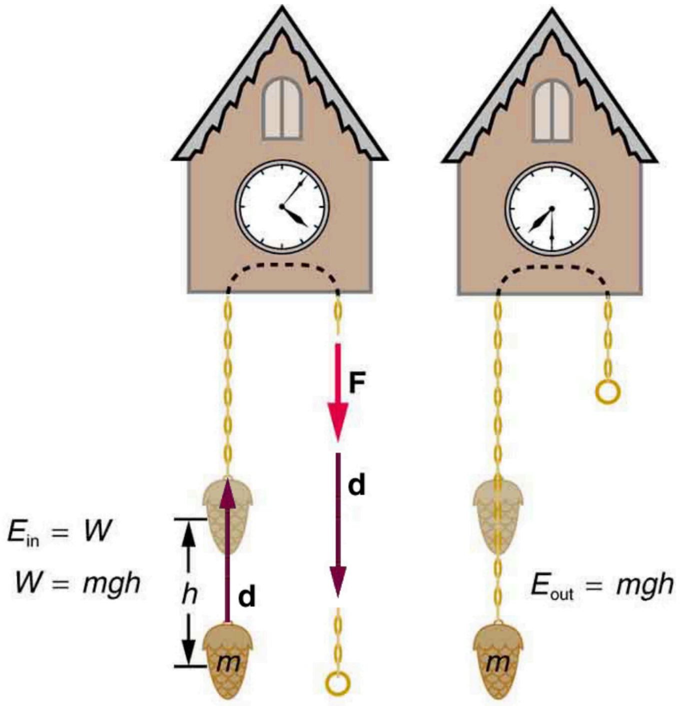


Gravitational potential energy

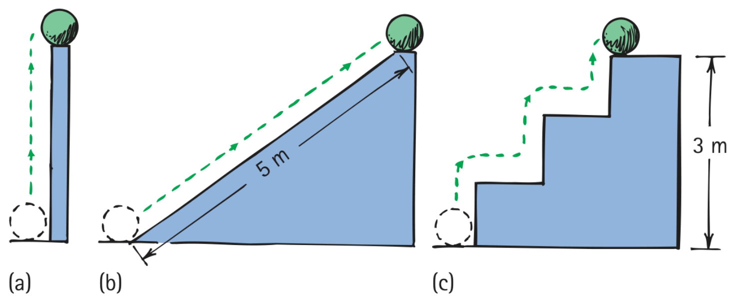
- Work is required to elevate objects against earth's gravity
- The potential energy due to elevated positions is called **gravitational potential energy** U_g
  - eg. water in an elevated reservoir 🌊 , rollercoaster at peak 🎢
- The amount of U_g possessed by an elevated object is equal to work W done against gravity in lifting it. Recall $W = Fd$ if only along the vertical, but F is weight $w = mg$ and d is height h, so

$$U_g = mgh$$

Example. If a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is



$$
\begin{aligned}
U_g &= mgh \\
&= (0.5 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m}) \\
&= 4.90 \text{ kg-m}^2/\text{s}^2 \\
&= 4.90 \text{ J}
\end{aligned}
$$

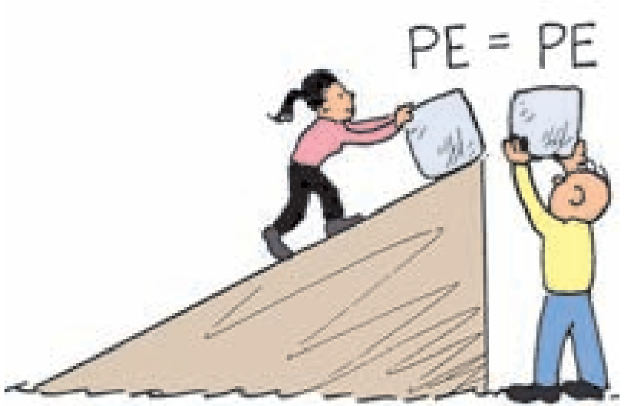


Example. The potential energy of 10-N ball is same in all cases,

$$U_g = (mg)h = (10\text{ N})(3\text{ m}) = 30\text{ J},$$

because work done in elevating it 3 m is the same whether it is (a) lifted with 10 N of force, (b) pushed with 6 N of force up the 5-m incline, or (c) lifted with 10 N up each 1-m stair. No work is done in moving the ball horizontally, if we ignore friction

Example. How much work is done in lifting the 100-N block of ice a vertical distance of 2 m? How much work is done in pushing the same block of ice up the 4-m-long ramp? (The force needed is only 50 N which is the reason ramps are used). What is the increase in the block's gravitational potential energy in each case?

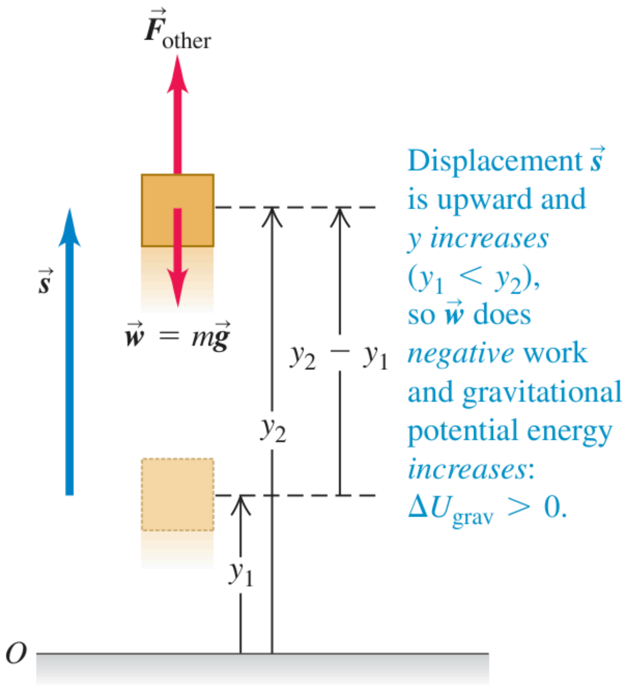


- $W = Fd = 100\text{ N} \times 2\text{ m} = 200\text{ J}$
- $W = Fd = 50\text{ N} \times 4\text{ m} = 200\text{ J}$
- In both cases, the block's U_g increases by 200 J. The ramp simply makes this work easier to perform

- Observe that weight and displacement are in opposite direction, so work W_g done on object by its weight is negative
- Thus, during displacement from y_1 to y_2,

$$W_g = U_{g,1} - U_{g,2} = -\left(U_{g,2} - U_{g,1}\right) = -\Delta U_g$$

- Negative sign in front of ΔU_g is essential. When object moves up, y increases, work done by gravity is negative ($W < 0$), and gravitational potential energy increases ($\Delta U_g > 0$)



Example. A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

- The work done on the person by the floor as he stops is

$$W = Fd\cos\theta = -Fd$$

  with a minus sign because displacement while stopping and force from floor are in opposite directions ($\cos 180° = -1$)

- But the work done on the person by its weight due to gravity is

$$W_g = -\Delta U_g = -mgh$$

- W is due to an upward force (normal force due to floor) while W_g is due to a downward force (weight due to gravity), so

$$W = -W_g$$

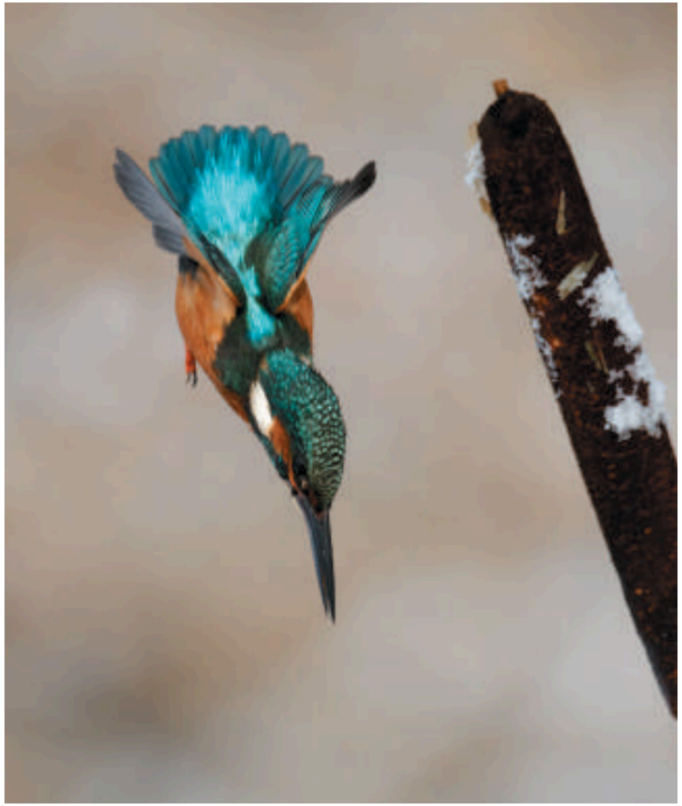
- Distance d that person's knees bend is much smaller than height h of the fall, so we ignore added change in U_g due to knee bend
- Equating these and computing for F exerted by floor, we get

$$W = -W_g \qquad \implies -Fd = -(-mgh) = mgh$$

$$\implies F = -\frac{mgh}{d} = \frac{(60 \text{ kg})(9.8 \text{ m/s}^2)(-3 \text{ m})}{5 \times 10^{-3} \text{ m}} = 3.53 \times 10^5 \text{ N}$$

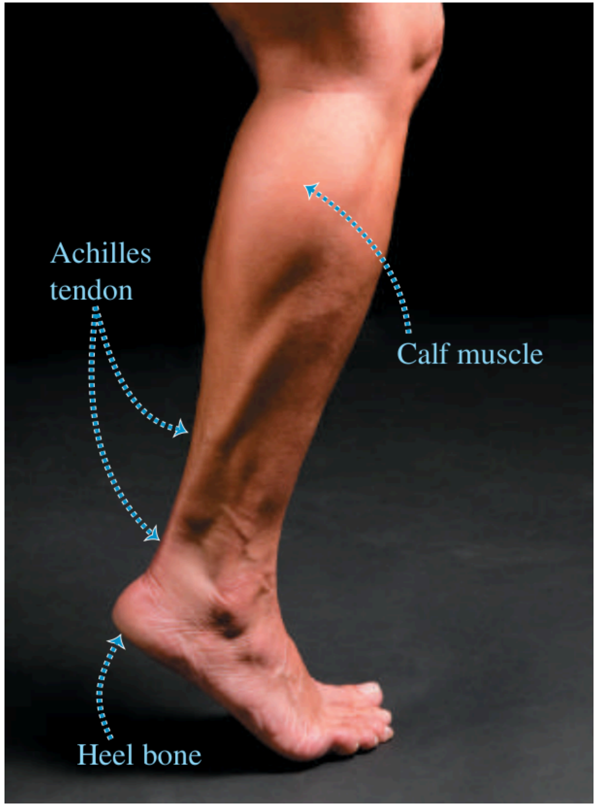
Quick aside: converting U_g to K

When a kingfisher spots a tasty fish, the bird dives from perch with its wings tucked in to minimize air resistance. The only force acting on the diving kingfisher is gravity, so the total mechanical energy is conserved: U_g lost as kingfisher descends is converted into the bird's K



Elastic yarn?

Achilles tendon, which runs along the back of ankle to heel bone, acts like a natural spring. When it stretches then relaxes, this tendon stores then releases elastic potential energy. This spring action reduces the amount of work your leg muscles must do as you run 🏃



Elastic potential energy

- An object is called **elastic** if it returns to its original shape and size after being deformed. We describe the process of storing energy in a deformable object such as a spring or rubber band in terms of **elastic potential energy** U_e
- Just as for gravitational work, we can express work done by the spring in terms of a quantity at the beginning and end of the displacement. This quantity is $\frac{1}{2}kx^2$, and we define it to be U_e:

$$U_e = \frac{1}{2}kx^2,$$

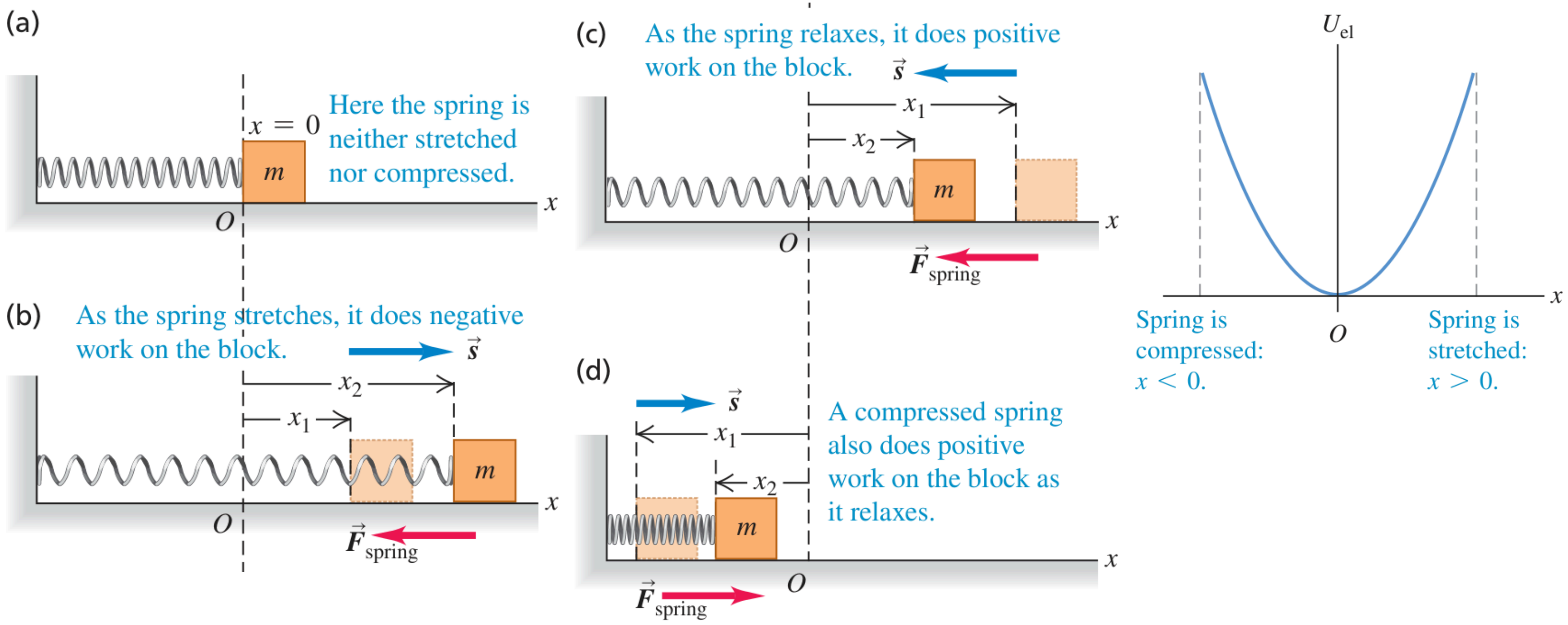
where k is the force constant of spring, and x is the elongation of spring ($x > 0$ if stretched, $x < 0$ if compressed)

- When a stretched spring is stretched farther, work W_e is negative ($W < 0$) and U_e increases ($\Delta U_e > 0$), that is more elastic potential energy is stored in the spring

$$W_e = U_{e,1} - U_{e,2} = -\Delta U_e$$

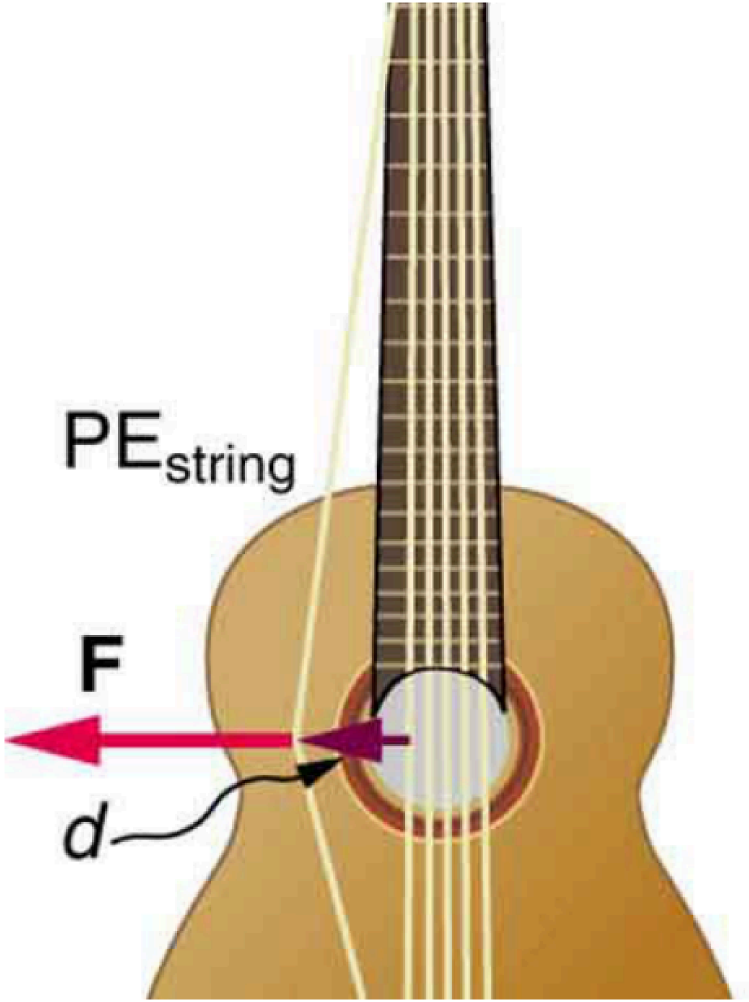
- Btw, recall that Hooke's law in elasticity lecture states that force magnitude F on spring and resulting deformation ΔL are proportional, as in $F = k\Delta L$. Here, we just replace ΔL with distance x that the spring is stretched or compressed

- U_e is positive for both positive and negative x values. The more a spring is compressed/stretched, the greater its elastic potential

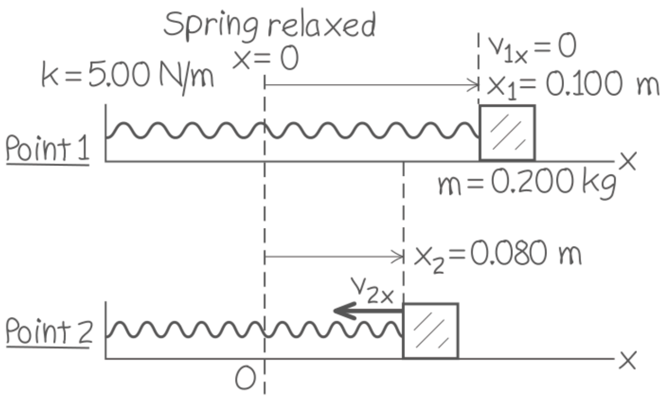


Example. Work is done to deform the guitar string, giving it potential energy U_e. When released, the U_e is converted to K and back to U_e as string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string 🔈

Btw here, $U_e = \text{PE}_{\text{string}}$



Example. A glider with mass $m = 0.200$ kg sits on a frictionless, horizontal air track, connected to a spring with spring constant $k = 5.00$ N/m. You pull on glider, stretching the spring 0.100 m, and release it from rest. The glider moves back to its equilibrium position $x = 0$. What is its potential energy at equilibrium? How about when $x = 0.080$ m?

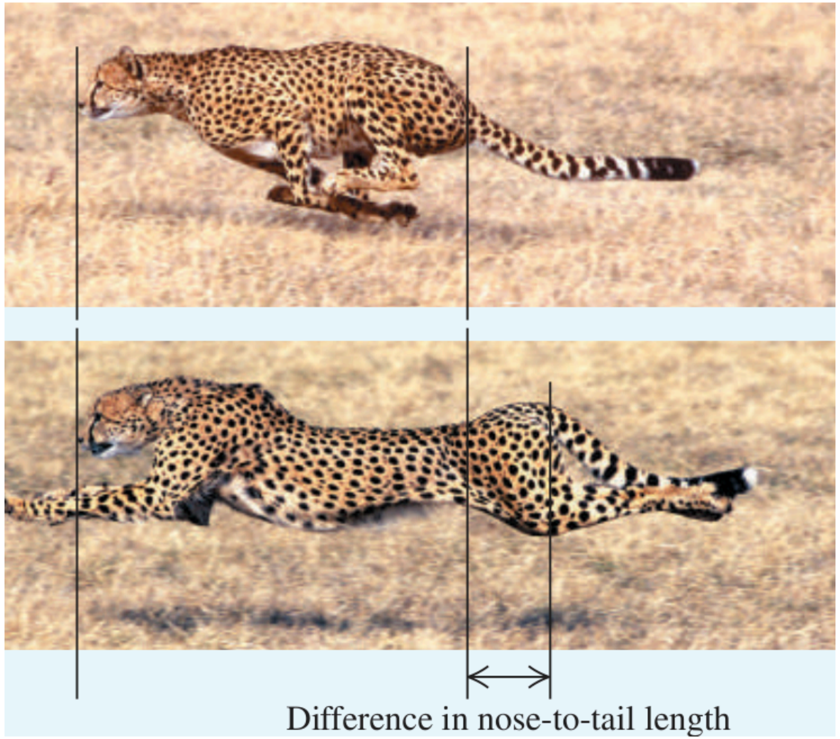


$$U_{e,1} = \frac{1}{2}(5.00 \text{ N/m})(0.100 \text{ m})^2 = 0.025 \text{ J}$$

$$U_{e,2} = \frac{1}{2}(5.00 \text{ N/m})(0.080 \text{ m})^2 = 0.016 \text{ J}$$

Quick aside: U_e of a cheetah

When cheetah gallops, its back flexes and extends. Flexion of the back stretches tendons and muscles along the top of the spine and also compresses the spine, storing U_e. When cheetah launches into its next bound, this energy is released, enabling cheetah to run more efficiently



Situations with both grav. U_g and elastic U_e

- What happens when we have both gravitational and elastic forces, such as block attached to hanging spring? And what if work is also done by forces that cannot be described by potential, eg. air resistance?
  $W_{\text{net}} = W_g + W_e + W_{\text{other}}$
  $W_{\text{net}} = \Delta K$ (work-energy th.)

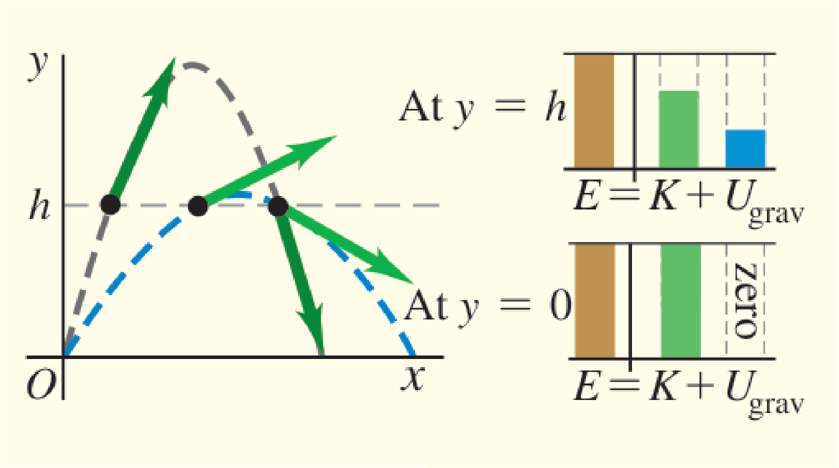
(Non-)conservative forces 🌀

Conservative

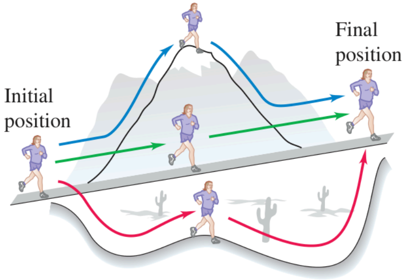
- In discussions of potential energy U, we talked about "storing" kinetic energy K by converting it to U, with the idea that we can retrieve it again as K
  - eg. stretching or compressing a spring 🪱
  - eg. throwing up ⚾ : it slows down as K becomes U_g, then speeds back up as it falls down as U_g converts back to K
- Here, there's a two-way conversion from kinetic to potential energy and back. In this case, the total mechanical energy E is said to be constant or **conserved** during the motion

Conservative

- A **conservative force** is a force that allows this two-way conversion between kinetic and potential energies
  - eg. gravitational force for U_g, elastic force for U_e



Because the gravitational force is conservative, the work it does is the same for all three paths.



Conservative

- Alternatively, it is such force which does work W that depend only on starting and ending points of motion, not on path taken
- When only conservative forces act on and within a system, total mechanical energy E, that is $E = K + U$, is constant as in

$$\Delta K + \Delta U = 0, \qquad \text{or}$$
$$K_1 + U_1 = K_2 + U_2$$

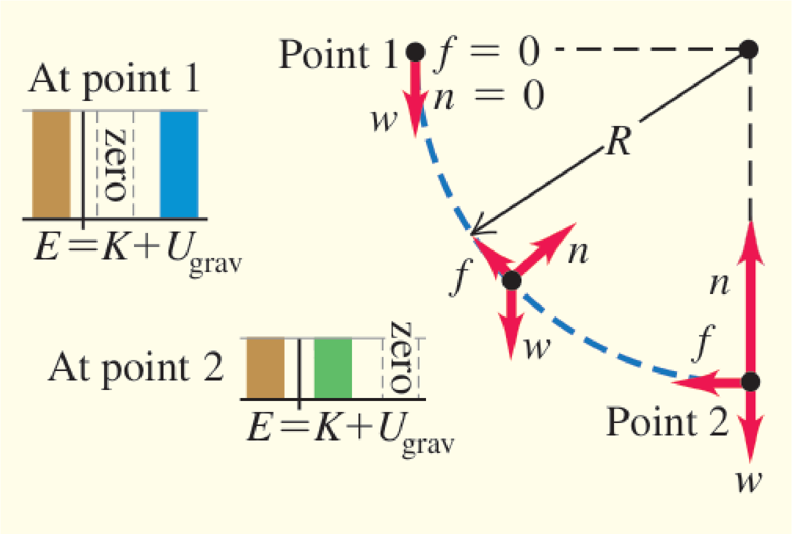
This is known as the **conservation of mechanical energy**

Nonconservative

- Not all forces are conservative. Consider the friction acting when a car with its brakes locked skids with decreasing speed (and decreasing K). The lost K can't be recovered by reversing the motion. Total mechanical energy E is **not conserved**

Nonconservative

- A **nonconservative force** is a force that doesn't allow the two-way conversion between kinetic and potential energies
  - eg. friction that changes mechanical energy into thermal energy



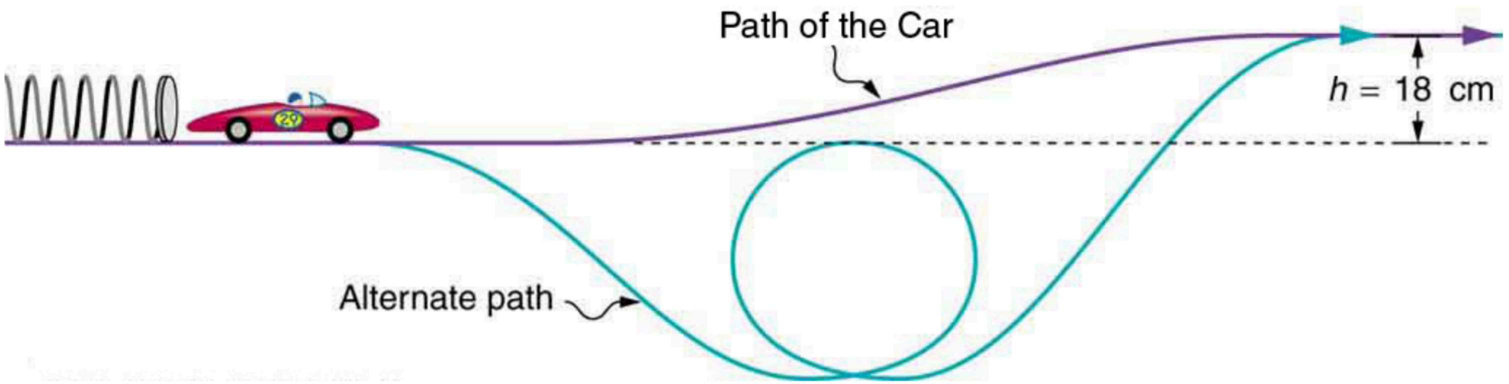
Nonconservative

- Alternatively, it is such force which does work W_{other} that depend on the path
- When nonconservative forces act on a system, total mechanical energy E, that is $E = K + U$, is not constant anymore. This is due to the work W_{other} done via the nonconservative forces dissipating some of the mechanical energy, so

$$\Delta K + \Delta U = W_{\text{other}}, \qquad \text{or}$$

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

Example. A 0.100-kg toy car is propelled by a compressed spring. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find how fast the car is going before it starts up the slope



- Elastic force and gravitational force are conservative, so conservation of mechanical energy can be used. So

$$\Delta K + \Delta U = 0$$
$$K_1 + U_1 = K_2 + U_2$$
$$\frac{1}{2}mv_1^2 + \left(mgh_1 + \frac{1}{2}kx_1^2\right) = \frac{1}{2}mv_2^2 + \left(mgh_2 + \frac{1}{2}kx_2^2\right)$$

- We're only concerned about the state of the car before it goes up the slope, so the initial and final heights h_1 and h_2 are zero
- Before car is released, it ain't moving, so initial speed v_1 is zero

- Final compression x_2 of spring is zero as it is unstretched then
- In other words, the initial elastic potential energy $U_{e,1}$ of the spring is completely converted to final kinetic energy K_2 in the absence of friction. Solving for final speed v_2, we get

$$\frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2$$

$$\implies v_2 = \sqrt{\frac{k}{m}}x_1 = \sqrt{\frac{250.0\ \text{N/m}}{0.100\ \text{kg}}}(0.400\ \text{m})$$

$$= 2.00\ \text{m/s}$$

Quick aside: nonconservative forces in a tire

A car tire deforms and flexes like a spring as it rolls, but it is not an ideal spring: internal friction forces act within rubber which results in mechanical energy being lost and converted to internal energy (also known as thermal energy). Thus temp. of a tire increases as it rolls

Conservation of energy ⚛️

Energy conservation

- When both conservative and nonconservative forces act, energy conservation can be applied to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws
- The **law of conservation of energy** states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same

- More profoundly, it states that energy is never created or destroyed, it only changes form. No exception to this rule has ever been found
- When all forms of energy are considered, we can write the law of conservation of energy as

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0, \qquad \text{or}$$
$$K_1 + U_1 - \Delta U_{\text{int}} = K_2 + U_2$$

where ΔU_{int} is the change in internal energy which encompasses the work W_{other} via nonconservative forces and the change in other forms of energy ΔE_{other}

- Commonly encountered other forms of energy E_{other} include electric, chemical, radiant, nuclear and thermal energies
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work, hence the W_{other} in above statement

Example. Does a car consume more fuel when its air conditioner is turned on? When its lights are on? When its radio is on while the motor is turned off in the parking lot?

- Yes to all, as all energy consumed ultimately comes from fuel. Even the energy taken from the battery must be given back to the battery by the alternator, which is turned by the engine, which runs from the energy of the fuel. There's no free lunch!

Example. The battery pack in this radio-controlled helicopter contains 2.4×10^4 J of electric energy. When this energy is used up, the internal energy of the battery pack decreases by this amount, so $\Delta U_{\text{int}} = -2.4 \times 10^4$ J

- This energy can be converted to kinetic energy to make the rotor blades and helicopter go faster, or to gravitational potential energy to make the helicopter climb

Machines

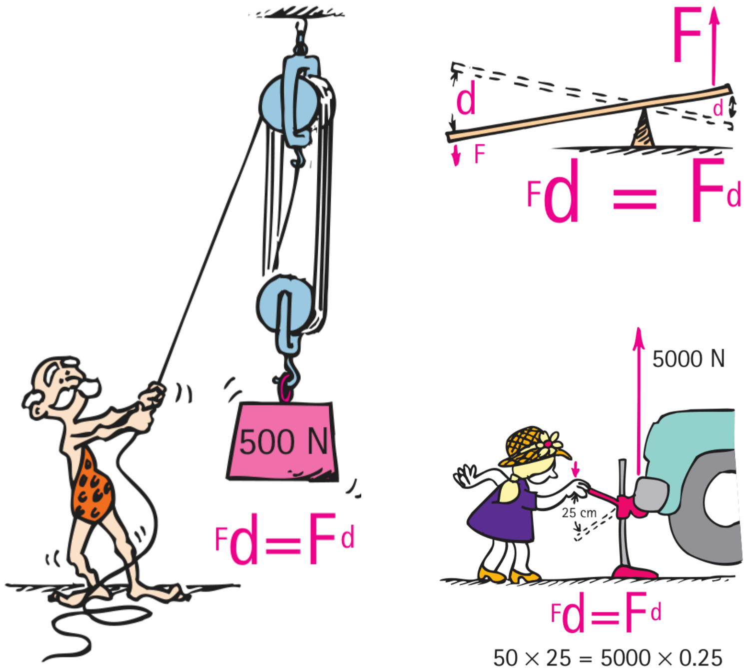
- A **machine** is a device that either multiples force or changes the direction of force. It exploits the conservation of energy
- If work done by friction or other nonconservative forces are small enough to ignore, then

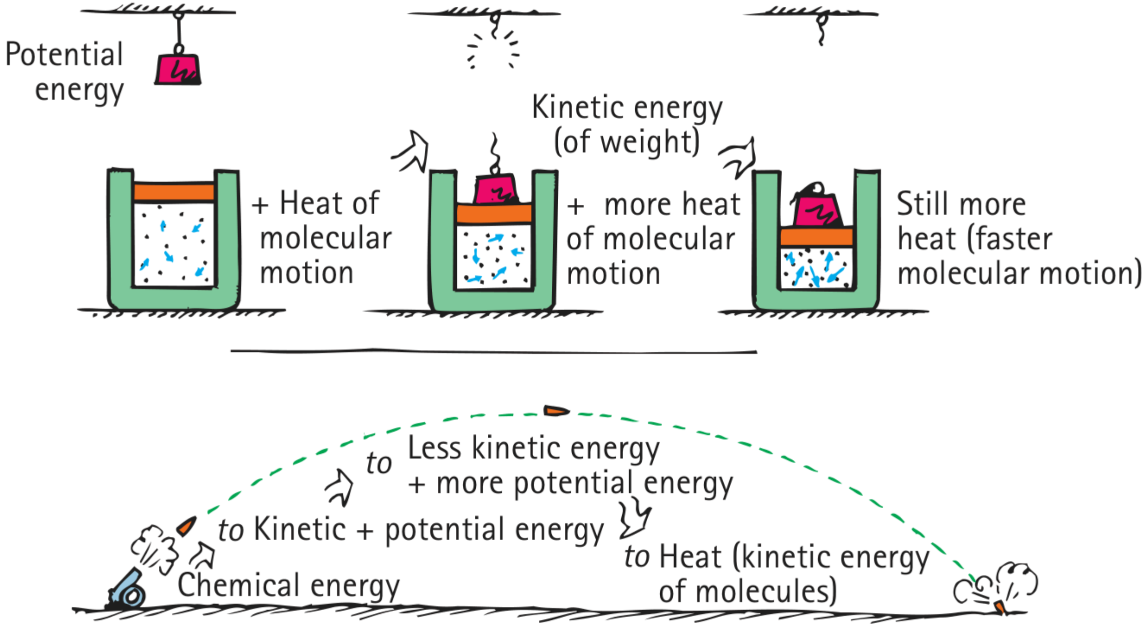
$$W_{\text{in}} = W_{\text{out}} \qquad \text{or} \qquad (Fd)_{\text{in}} = (Fd)_{\text{out}}$$

- The **efficiency** ε of a machine or human is defined as

$$\varepsilon = W_{\text{out}}/E_{\text{in}},$$

where W_{out} is useful work output and E_{in} is energy consumed

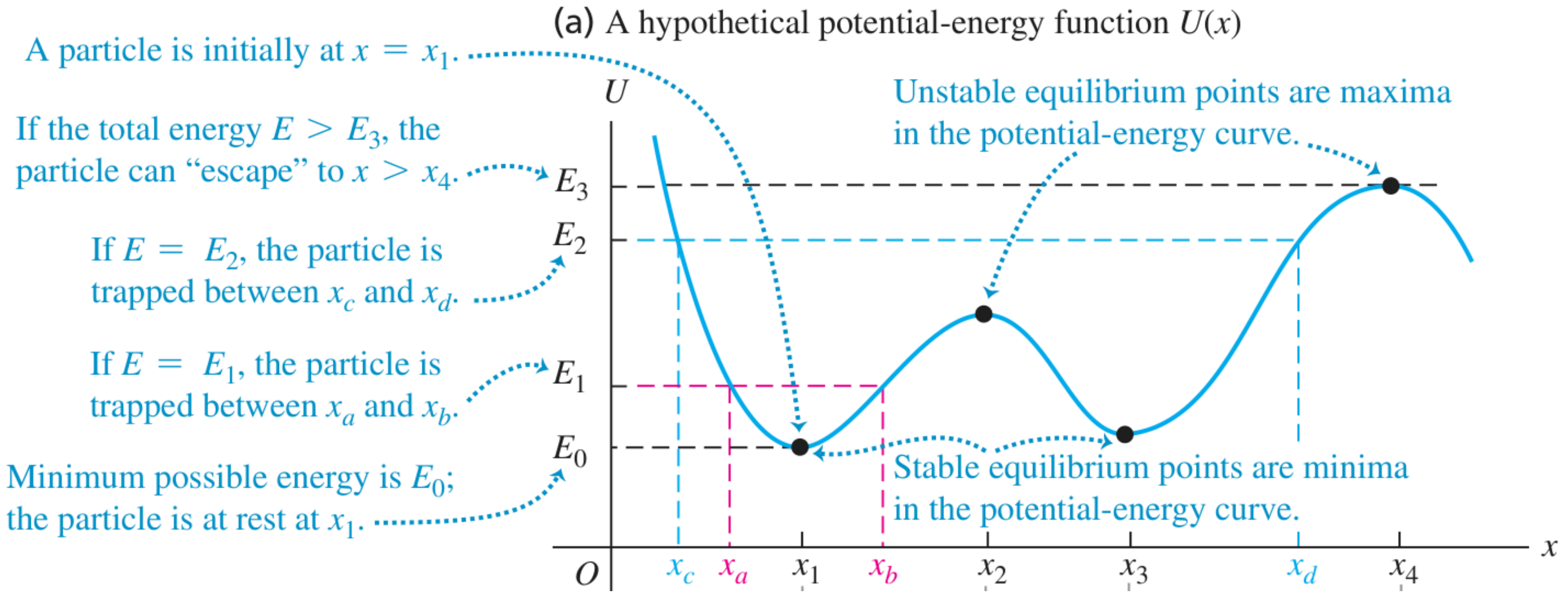




- Here, some energy goes out in the hot exhaust gases, and some is dissipated to the air through the cooling system or directly from hot engine parts, making the engine a little bit inefficient

Energy diagrams

- Any minimum in a potential energy diagram is a **stable equilibrium** position. Any maximum is **unstable equilibrium**



Quick aside: acrobats in equilibrium

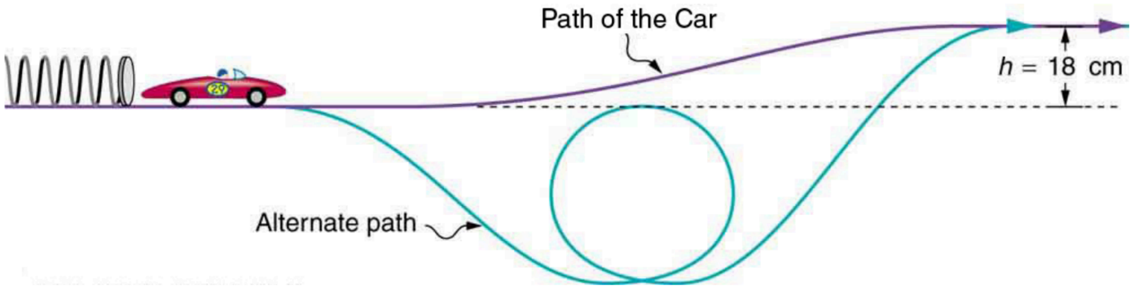
Each of these acrobats is in unstable equilibrium. The gravitational potential energy is lower no matter which way an acrobat tips, so if she begins to fall she will keep on falling. Staying balanced requires the acrobats' constant attention.

Quiz time 🕐

Spring-propelled car, again 🚗💨🪱

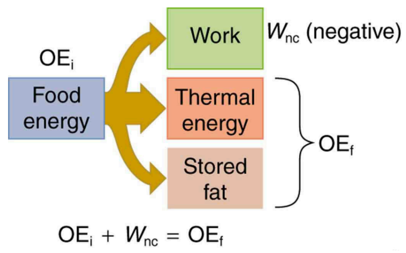
Going back to the spring-propelled car example above, how fast is it going at the top of the slope?

Recall that a 0.100-kg toy car is propelled by a compressed spring. The car follows a track that rises 0.180 m above the starting point. Spring is compressed 4.00 cm and has force constant of 250.0 N/m



Work, energy, and power in humans 🧑

On humans

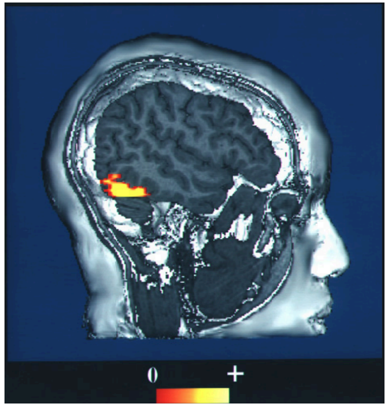


- Human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue

- The rate at which body uses food energy to sustain life and to do different activities is called the **metabolic rate**, and the corresponding rate when at rest is called **basal metabolic rate**
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next

Organ	Power consumed at rest (W)	O_2 consumption (mL/min)	BMR (%)
liver & spleen	23	67	27
brain	16	47	19
skeletal muscle	15	45	18
kidney	9	26	10
heart	6	17	7
other	16	48	19
total	85	250	100

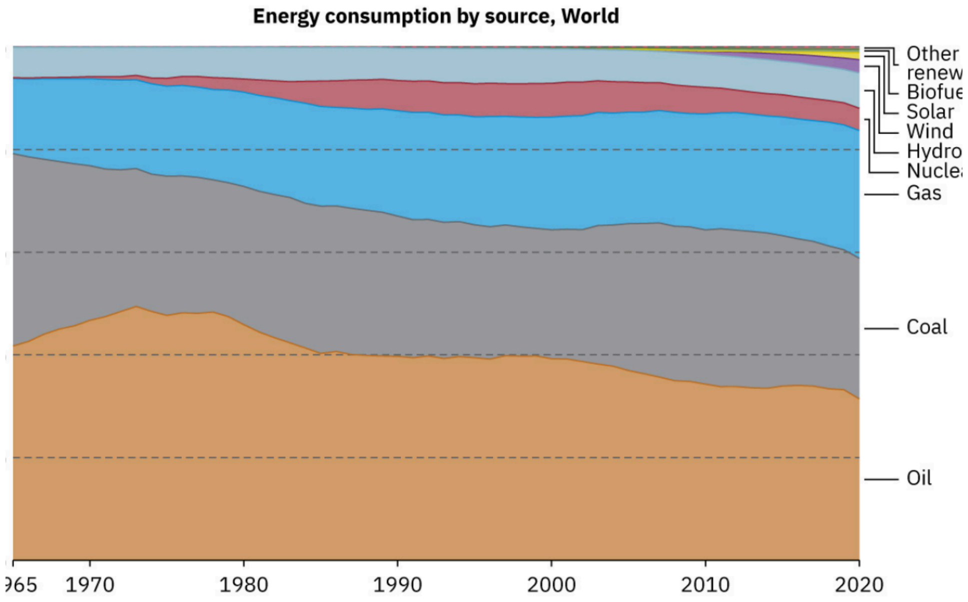
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food



- This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, patient was being asked to recognize faces

On world energy use

- The relative use of different fuels to provide energy has changed over years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing



On world energy use

- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources. The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by gross domestic product per capita, are matched by higher levels of energy consumption per capita

- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes