Work and energy

R. Torres

2025 W41[1]

[1] Phys 20.01 Mod 3. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Work 💼

Power ⚡

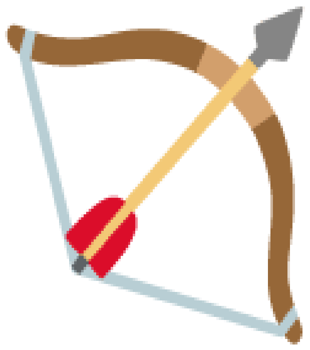
Work-energy theorem 📚

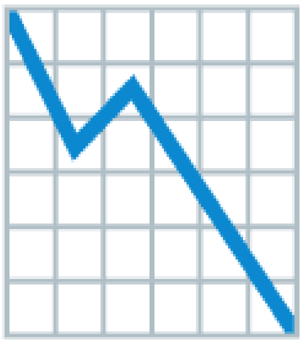
Quiz time 🕐

Work 💼

If Newton can't, werk can

- Suppose you try to find the speed of an arrow that has been shot from a bow
- You apply Newton's laws to no avail. Blocker: after arrow is released, bow string exerts varying force dep. on position
- Fear not, we introduce a new method via work and energy





Work

- It's hard work to pull heavy 🛋️ across room, to lift stack of 📚 from floor to shelf, or to push stalled 🚗 off road
- Indeed, all of these examples agree with everyday meaning of "work", ie. any activity that requires muscular or mental effort

Work

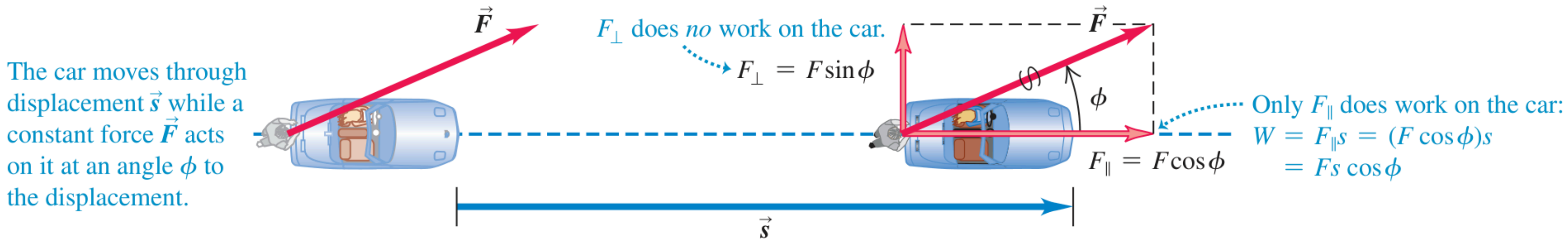
- In physics, work has much more precise definition. Using this we find that no matter how complicated motion is, we can describe it in terms of work done on object and its kinetic energy $(m, \vec{v})$
- Previous examples 🛋️ 📚 🚗 have something in common: you do work by exerting a force on an object while that object moves from one place to another, ie. as it undergoes a *displacement*
- You also do *more work* if force is greater (push harder on car) or if displacement is greater (push car farther down the road)

FIGURE 7.1
Compared with the work done in lifting a load of gravel one story high, twice as much work is done in lifting the same load two stories high. Twice the work is done because the distance is twice as great.

FIGURE 7.2
When twice the load of gravel is lifted to the same height, twice as much work is done because the force needed to lift it is twice as much.

- Simply, **work** is the transfer of energy by a force acting on an object as it is displaced. More precisely, work W that a constant force $\vec{F}$ does on an object during a displacement d is product of magnitude d of displacement and magnitude $F_{\parallel}$ of parallel component of force, as in

$$W = \vec{F} \cdot \vec{d} = F_{\parallel} d = (F \cos\theta) d = F d \cos\theta$$



- If the force applied is in the same direction as displacement, as in when angle $\theta = 0$, then work is just

$$W = Fd \cdot \cos 0° = Fd \cdot 1 = Fd$$

- The SI unit for work and energy is the joule (J), where 1 J = 1 N m = 1 kg m^2/s^2

- Some notes
  - Don't confuse uppercase W (work) with lowercase w (weight)
  - Work is a scalar, even though it's calculated from two vectors.

    eg. (5 N force ➡️ on object moving 6 m to east) does the same work as (5 N force ⬆️ on object moving 6 m to north)

Example. How much work is needed to lift a bag of groceries that weighs 200 N to a height of 3 m? How much work is needed to lift the bag twice as high? 🛒

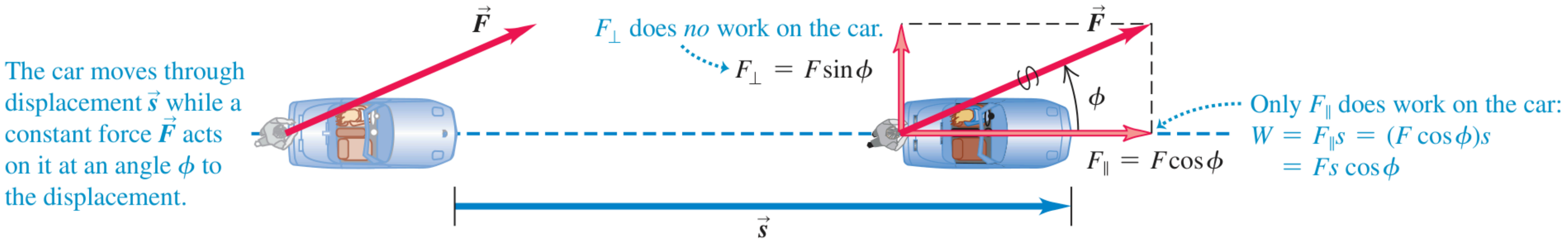
- Since the force exerted on the bag is in the same direction of the bag's motion, we can just use $W = Fd$ to calculate work

$$W = Fd = 200 \text{ N} \times 3 \text{ m} = 600 \text{ J}$$

- Lifting the bag twice as high requires twice the work

$$W = Fd = 200 \text{ N} \times 6 \text{ m} = 1200 \text{ J}$$

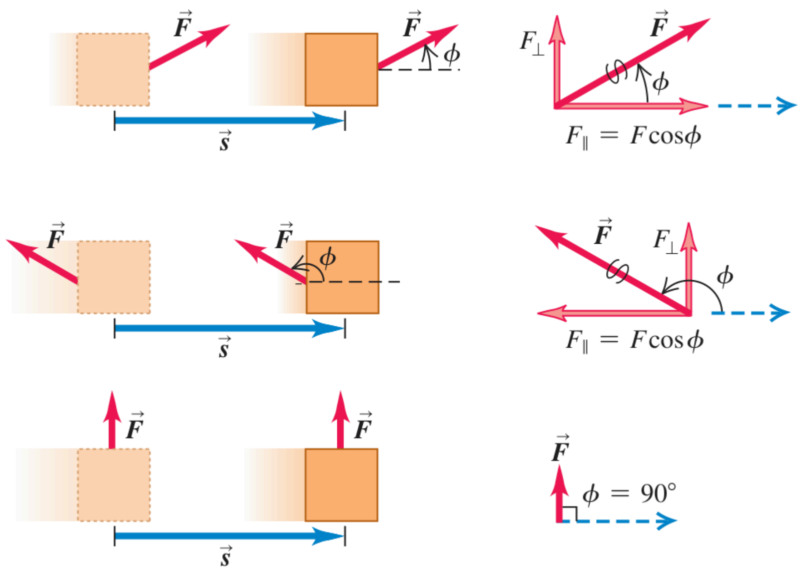
Example. Steve exerts a steady force of magnitude 210 N on the stalled car as he pushes it a distance of 18 m. The car also has a flat tire, so to make the car track straight Steve must push at angle of 30° to direction of motion. How much work does Steve do? 🚗

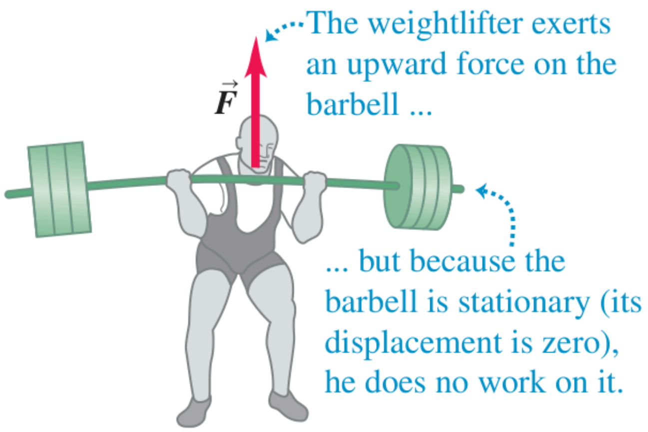


$$W = Fd\cos\theta = (210\ \text{N})(18\ \text{m})\cos 30^\circ = 3.3 \times 10^3\ \text{J}$$

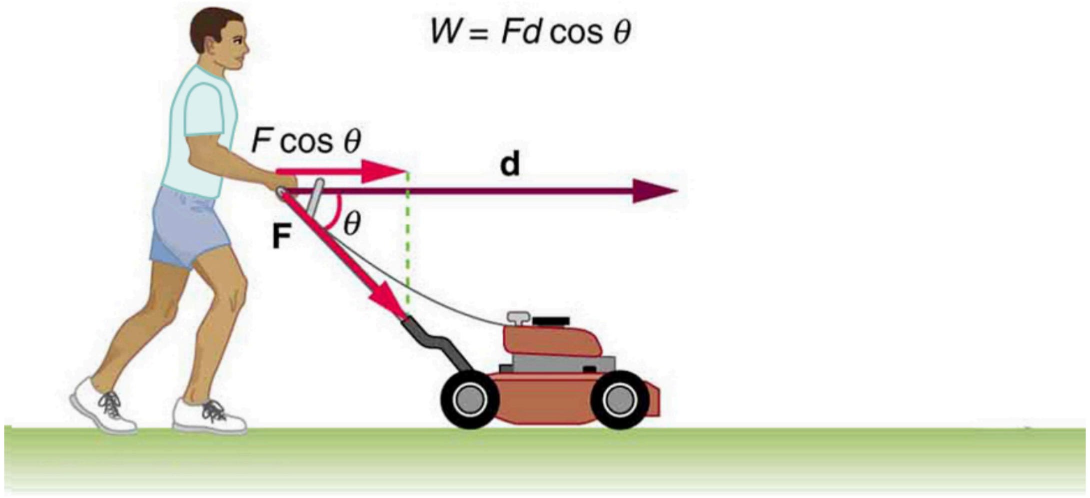
Work: positive, negative, or zero

- Work can also be negative or zero. This is the essential way in which work as defined in physics differs from the "everyday" definition of work
- If $\vec{F}$ has component in direction of displacement d, W is positive
- If $\vec{F}$ has component opposite to direction of d, then W is negative because $F \cos\theta$ is negative for $90° < \theta < 180°$
- If $\vec{F}$ is perpendicular to direction of d, W is zero because force does no work on object

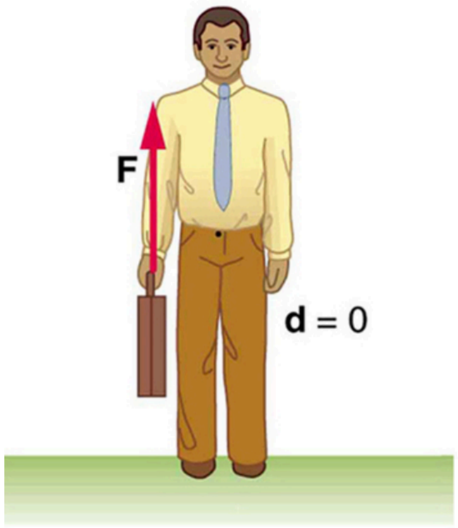


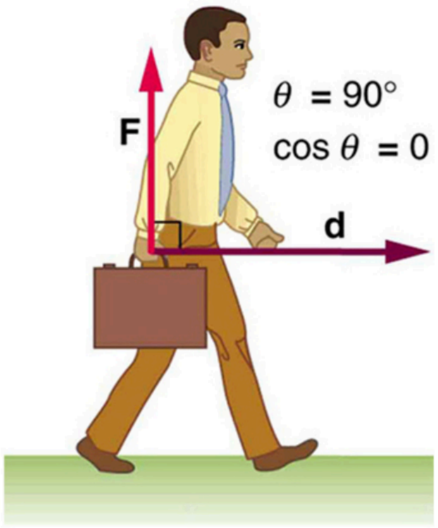


- Restating, work done by a force is zero if displacement is either zero or perpendicular to force. Otherwise, work is positive if force and displacement have same direction, and negative if they have opposite direction

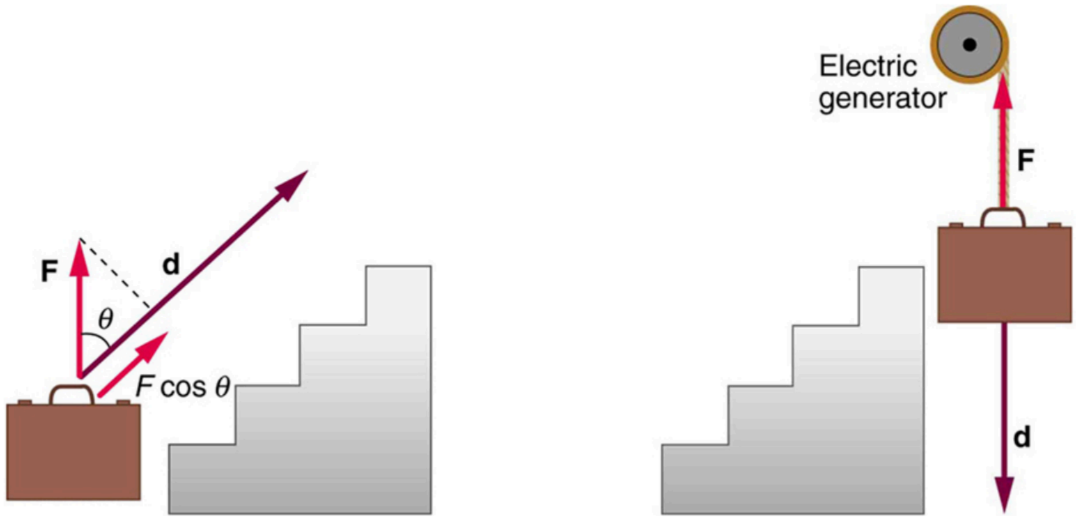


- Here, the work done by force $\vec{F}$ on this lawn mower is $Fd\cos\theta$. Note that $Fd\cos\theta$ is component of force in direction of motion





- Here, a person holding a 💼 does no work on it, because there is no displacement. No energy is transferred to or from the 💼
- The person moving the 💼 horizontally at a constant speed does no work on it, and transfers no energy to it



- Here, work is done on the 💼 by carrying it up stairs at constant speed, because there is necessarily a component of force $\vec{F}$ in the direction of the motion. Energy is transferred to the 💼 and could in turn be used to do work

- When 💼 is lowered, energy is transferred out of the 💼 and into an electric generator. Here the work done on the 💼 by the generator is negative, removing energy from the 💼, because $\vec{F}$ and $\vec{d}$ are in opposite directions

Total work

- The net work $\sum W = W_{\text{net}}$ is the work done by the net force acting on an object
  - One way to calculate this is to compute work done by each separate force acting on an object and get the algebraic sum

$$W_{\text{net}} = \sum W = W_1 + W_2 + \cdots = F_1 d_1 \cos\theta_1 + F_2 d_2 \cos\theta_2 + \cdots$$

  - Another way is to compute for the vector sum of the forces $F = \left|\vec{F}\right| = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$ then use it directly with $W = Fd$

Quick aside: work and muscle fibers 🦾

The fiberlike cells of skeletal muscle can shorten, causing muscle to contract and exert force on tendons to which it attaches. Muscle can exert ~ 0.3 N/mm^2 of cross-sectional area: the greater c-s area, more fibers muscle has and more force it can exert when it contracts

Power ⚡

Same work done, but more tired?

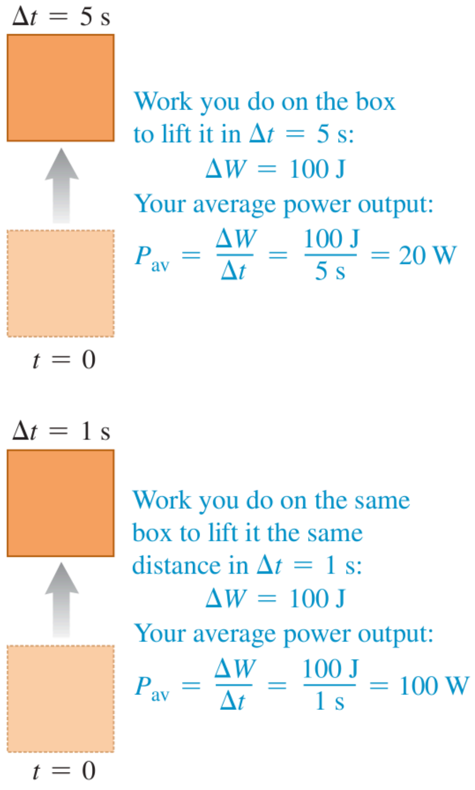
- The definition of work says nothing about how long it takes to do work. We do the same amount of work when we carry a load of groceries up a flight of stairs whether we walk up or run up
  - So why are we more tired after running upstairs 🏃 in a few seconds than after walking upstairs 🚶 in a few minutes? 😳

Power

- **Power** is rate at which work is done. Average power is

$$P = \Delta W / \Delta t$$

- The SI unit for power is the watt (W), where 1 W = 1 J/s
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where 1 hp = 746 W



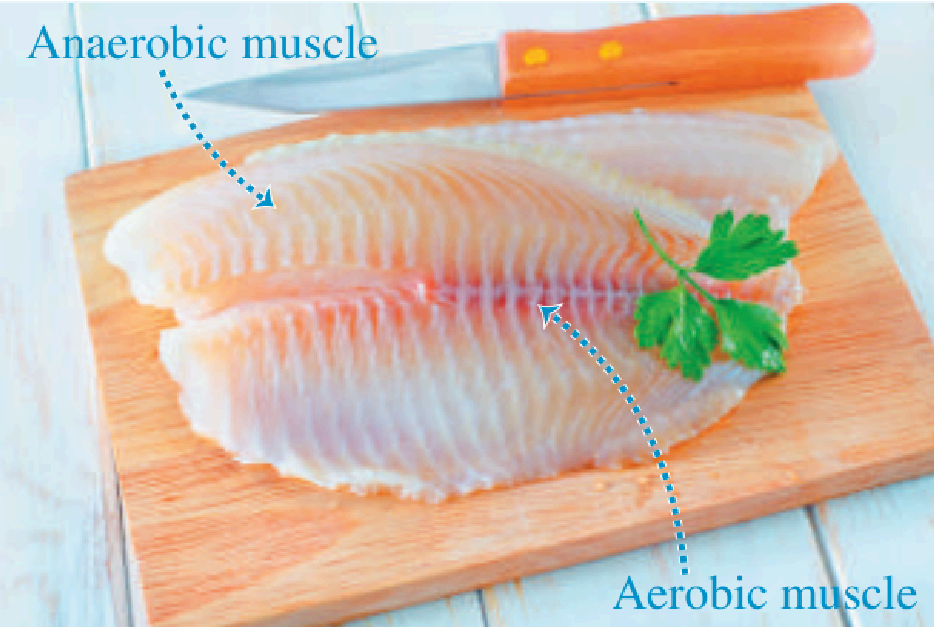
Power

Example. If a new-model forklift has twice the power of an older one, how much more load can it lift in same amount of time? If it lifts the same load as old one, how much faster can it operate? 🚚

- The new forklift delivers twice the power, so it can lift twice the load in the same time or the same load in half the time
- Either way, the owner of the new forklift is happy 😀

Quick aside: muscle power ⚡

- Skeletal muscles provide the power that makes animals move
- Muscle fibers that rely on anaerobic metabolism do not require oxygen, they produce large amounts of power but are useful for short sprints only



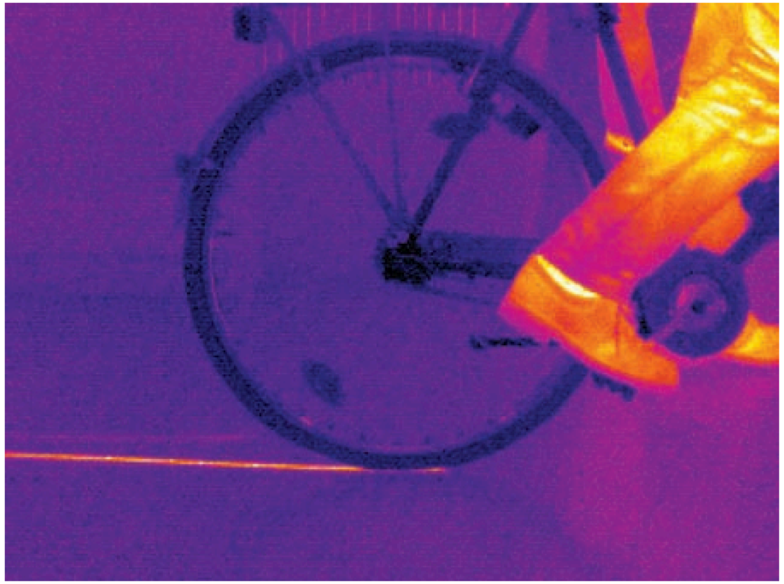
Quick aside: muscle power ⚡

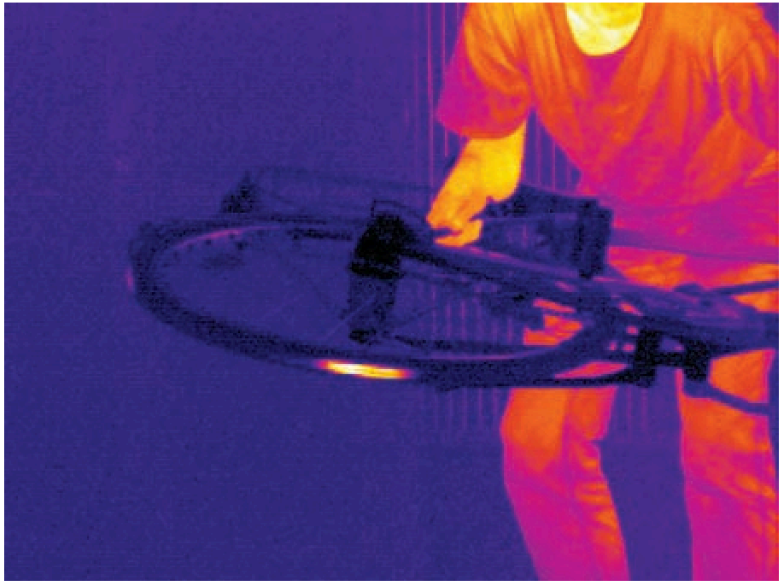
- Muscle fibers that metabolize aerobically use oxygen and produce smaller amounts of power for long intervals
- Both fiber types are visible in a fish fillet: the pale (anaerobic) muscle is used for brief bursts of speed, while the darker (aerobic) muscle is used for sustained swimming 🐟

Work-energy theorem 📚

Work transfers energy

- When work is done by an archer in drawing a bowstring, the bent bow acquires the ability to do work on the arrow ✓
  - When work is done to wind a spring mechanism, the spring acquires ability to do work on various gears to run a clock, ring a bell, or sound an alarm ✓
  - etc.
- In each case, something has been acquired that enables object to do work. This something is *energy*
- Basically, work done on an object transfers energy to the object





- Due to friction, energy is transferred both into the floor and into the tire when bicycle skids to a stop. An infrared camera reveals the heated tire track (the red streak on the floor, left) and the warmth of the tire (right).

- Energy appears in many forms eg. chemical, electrical, etc. But for now, we focus on two most common forms of **mechanical energy**: the energy due to the position of something (potential) and the movement of something (kinetic), or their sum
- The translational **kinetic energy** K of an object of mass m moving at speed v is

$$K = \frac{1}{2}mv^2$$

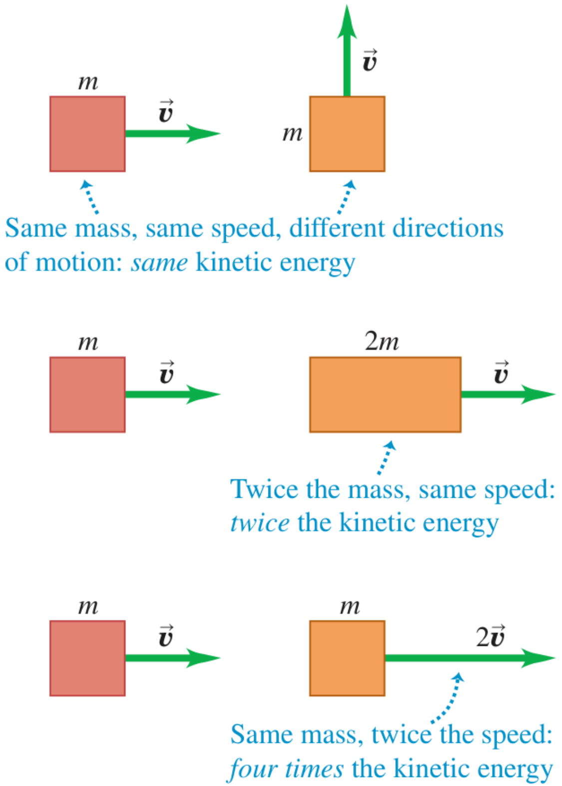
  - Like work, this is a scalar and is measured in SI unit joules

Work-energy theorem

- The **work-energy theorem** states that the net work W_{net} on a system changes its kinetic energy, as in

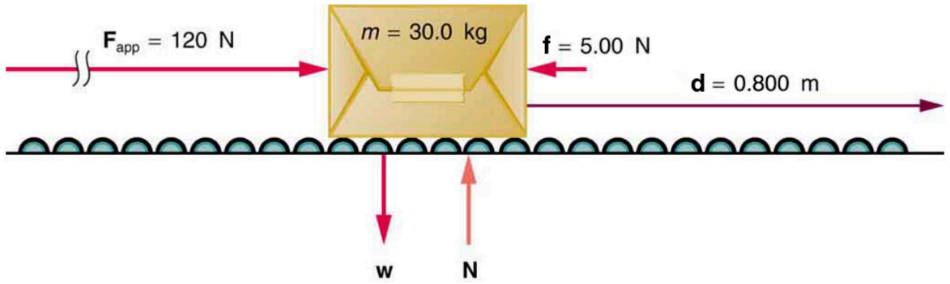
$$W_{\text{net}} = \Delta K = K_2 - K_1$$

  - This tells us only about changes in speed, not $\vec{v}$, since K is scalar (no dir.)



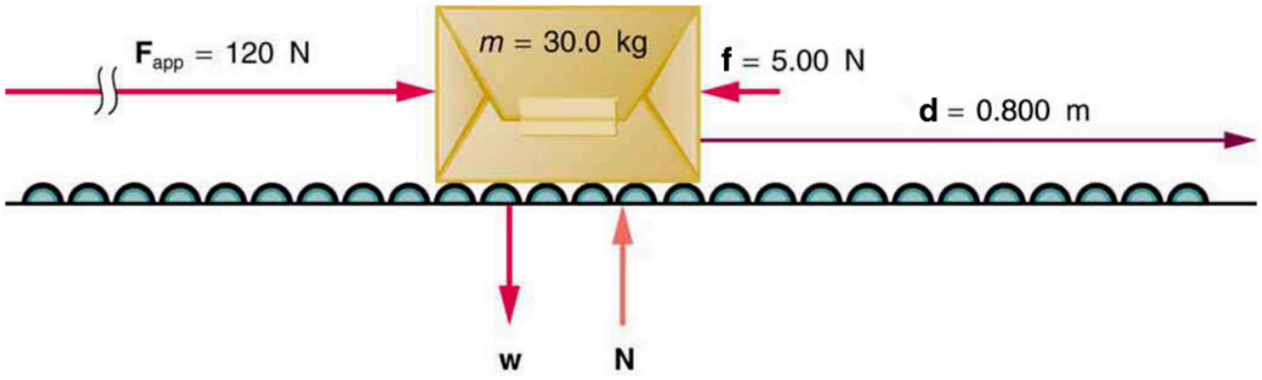
Work-energy theorem

Example. Suppose a 30.0-kg package on the roller belt conveyor system is moving at 0.500 m/s. What is its kinetic energy?



- $K = \frac{1}{2}mv^2 = \frac{1}{2}(30.0\text{ kg})(0.500\text{ m/s})^2 = 3.75\text{ J}$
- Btw, the force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work

Example. Suppose that you push on the 30.0-kg package in the roller belt conveyor example above with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N. (a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.



- (a) The net force acting on the package is push force and friction, as in $F_{\text{net}} = \sum F = 120\text{ N} + (-5\text{ N}) = 115\text{ N}$. Thus,

$$W_{\text{net}} = \sum W = \left(\sum F\right)d = (115\text{ N})(0.800\text{ m}) = 92.0\text{ N-m} = 92.0\text{ J}$$

  - The person actually does more work than this to overcome the friction that opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy
- (b) Forces acting on the package are gravity, normal force, friction force, and the applied force. Normal force and gravity are each perpendicular to displacement and thus do no work.

- The applied force does work

$$W_{\text{app}} = F_{\text{app}} d \cos 0° = F_{\text{app}} d = (120\ \text{N})(0.800\ \text{m}) = 96.0\ \text{J}$$

- Friction force and displacement are in opposite directions, so $\theta = 180°$, and work done by friction is

$$W_f = F_f d \cos 180° = -F_f d = -(5.00\ \text{N})(0.800\ \text{m}) = -4.00\ \text{J}$$

- So amounts of work done by each force acting on package are

$$W_g = 0, \quad W_N = 0, \quad W_{\text{app}} = 96.0\ \text{J}, \quad W_f = -4.00\ \text{J},$$

$$\implies W_{\text{net}} = \sum W = W_g + W_N + W_{\text{app}} + W_f = 92.0\ \text{J}$$

Example. Find the speed of the package the same conveyor example at the end of the push, using work and energy concepts.

- The work-energy theorem is

$$W_{\text{net}} = \Delta K = K - K_0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\Longleftrightarrow \frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0^2$$

But in the previous two examples, we already computed for W_{net} and initial kinetic energy $K_0 = \frac{1}{2}mv_0^2$, thus

$$\frac{1}{2}mv^2 = 92.0\ \text{J} + 3.75\ \text{J} = 95.75\ \text{J}$$

which solving for final speed v gives

$$v = \sqrt{\frac{2(95.75\ \text{J})}{m}} = \sqrt{\frac{191.5\ \text{kg-m}^2/\text{s}^2}{30.0\ \text{kg}}} = 2.53\ \text{m/s}$$

- Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package. ⚡

Checkpoint. Can objects have energy? ⚡

Can objects have work? 💼

Checkpoint. Can objects have energy? ⚡

Can objects have work? 💼

Yes, an object can have energy, eg. chemical, kinetic, thermal.

Checkpoint. Can objects have energy? ⚡

Can objects have work? 💼

Yes, an object can have energy, eg. chemical, kinetic, thermal. No, an object cannot have work. Work is not something an object *has*, rather it's energy in transit

Quiz time 🕐

Pitch that ball 🦇 🏏

A baseball pitcher does work with his throwing arm to give the ball a property called kinetic energy, which depends on the ball's mass and speed. Which has the greatest kinetic energy?

- a ⚾ of mass 0.145 kg moving at 20.0 m/s
- a smaller ⚾ of mass 0.0145 kg moving at 200 m/s
- a larger ⚾ of mass 1.45 kg moving at 2.00 m/s
- all three ⚾ ⚾ ⚾ have the same kinetic energy
- it depends on the directions in which the ⚾ moves