Forces and laws of motion: applications

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2025 W37–38[1]

[1]Phys 20.01 Mod 2. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Previously ⏪

Some preliminaries 📕

Applying Newton's laws ⚖️

Quiz time 🕐

Previously ⏪

Forces, laws of motion

First law: which one?

Example. In which case is there zero net external force on object?

- ✈️ flying due north at steady 120 m/s and at constant altitude
- 🚗 driving straight up hill with 3° slope at constant 90 km/h
- 🐦 circling at constant 20 km/h at constant height of 15 m
- 📦 with slick, frictionless surfaces in the back of a truck as the truck accelerates forward on a level road at 5 m/s^2

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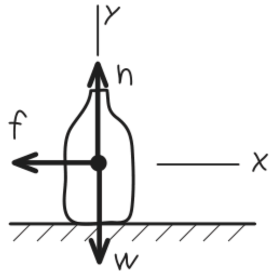
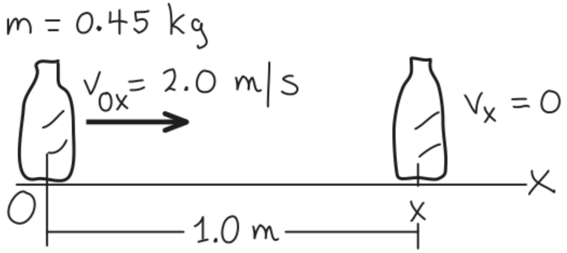
- Therefore answer is ✈️ 🚗 📦

Second law: ketchup later?

Example. A waitress shoves a ketchup bottle with mass 0.45 kg to her right along a smooth, level lunch counter. The bottle leaves her hand moving at 2.0 m/s, then slows down as it slides because of a constant horizontal friction force exerted on it by the countertop. It slides for 1.0 m before coming to rest. What are the magnitude and direction of the friction force acting on the bottle?

Second law: ketchup later?





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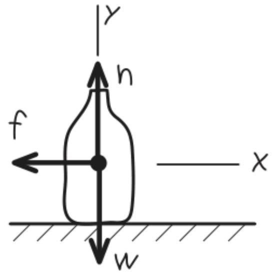

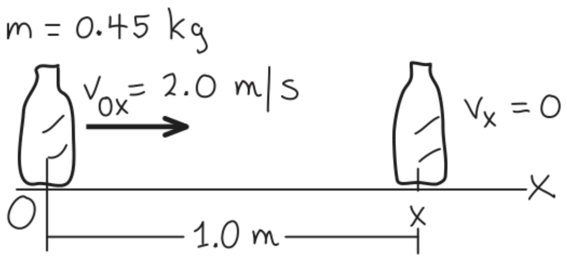

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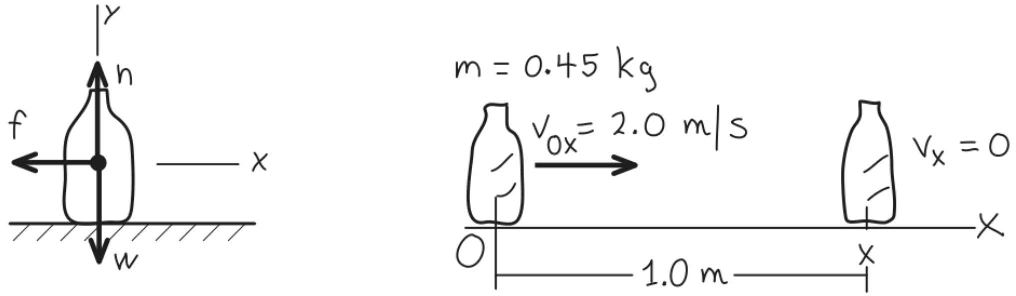

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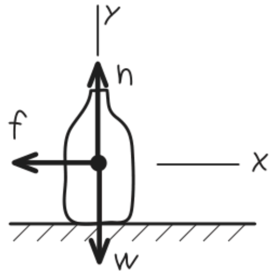

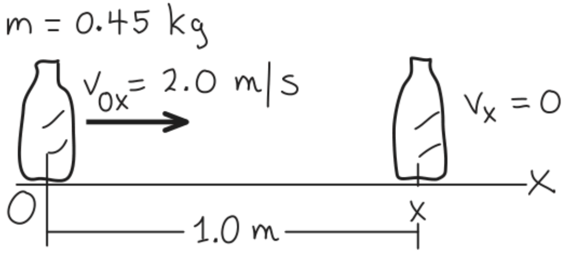

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$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(0 \text{ m/s})^2 - (2.0 \text{ m/s})^2}{2(1.0 \text{ m} - 0 \text{ m})} = -2.0\text{m/s}^2$$

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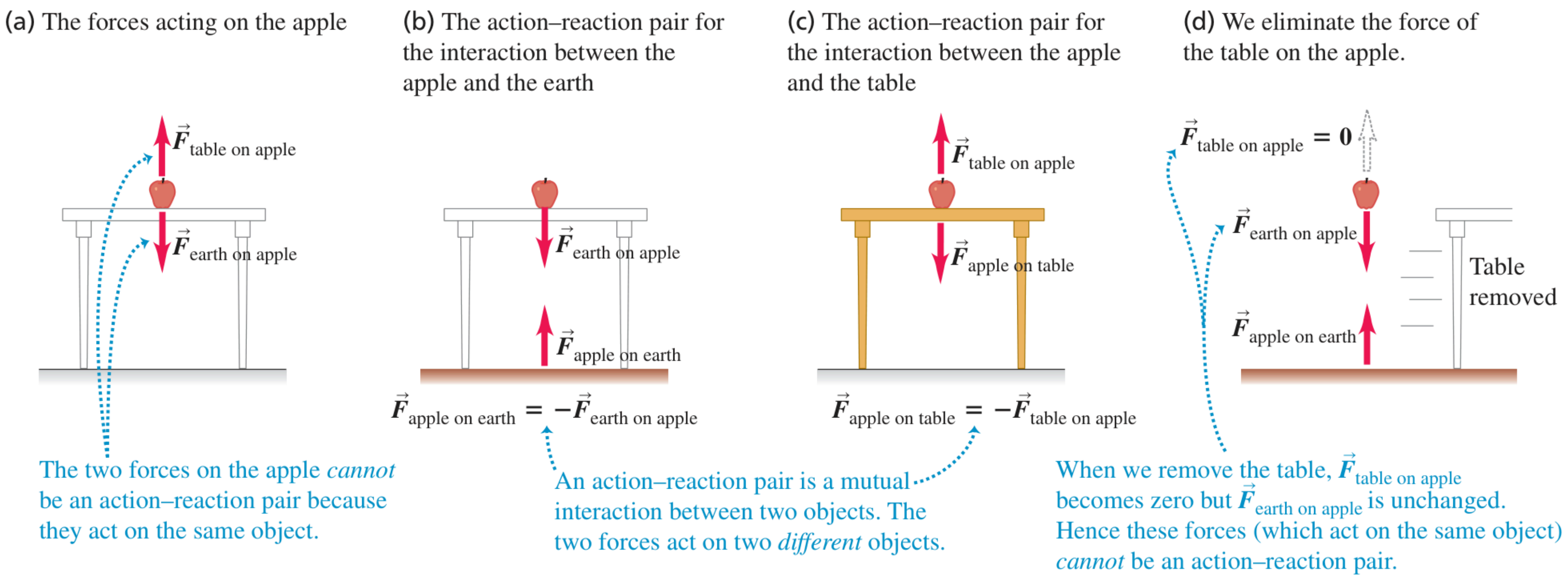
- The negative sign shows that net external force on the bottle is towards left. The magnitude of friction force is then $f = 0.90$ N

Third law: awe dropping?

Example. An apple sits at rest on a table, in equilibrium. What forces act on the apple? What is the reaction force to each of the forces acting on the apple? What are the action-reaction pairs?



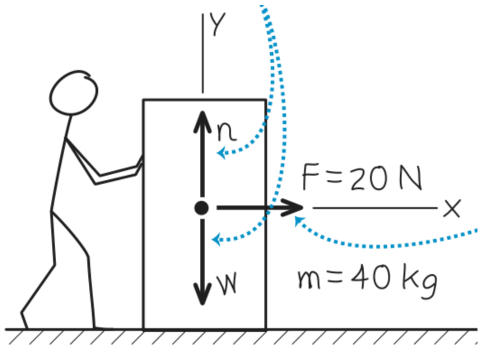
- The two forces in the action-reaction pair always act on two different objects

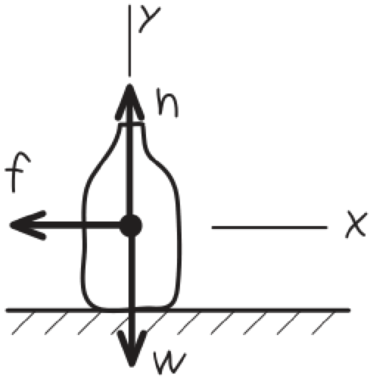
Questions? 🤓

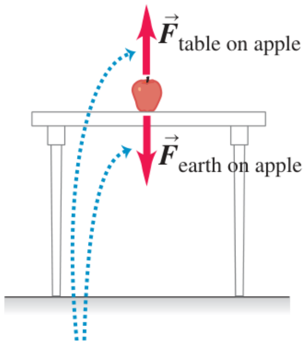
Some preliminaries 📕

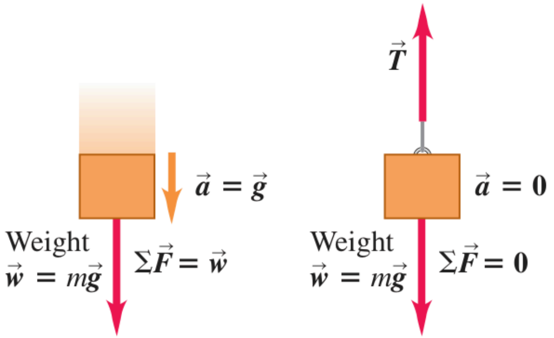
Free-body diagram

- A **free-body diagram** shows the object we choose to analyze by itself, "free" of its surroundings, with vectors drawn to show magnitudes and directions of all forces that act on the object
  - Here, body is another word for object

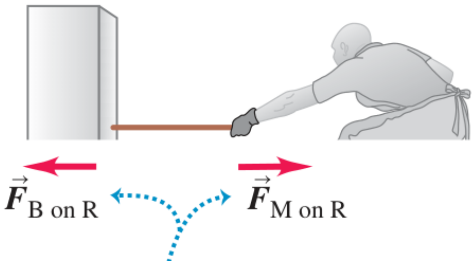


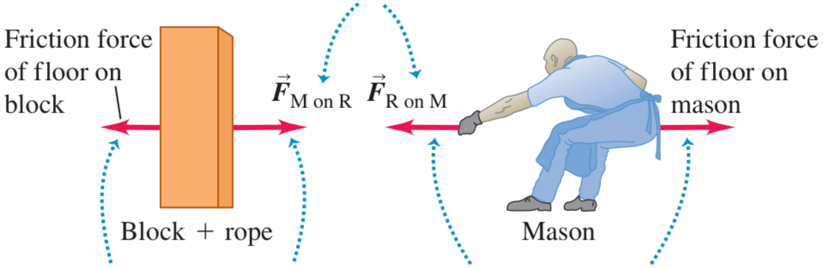




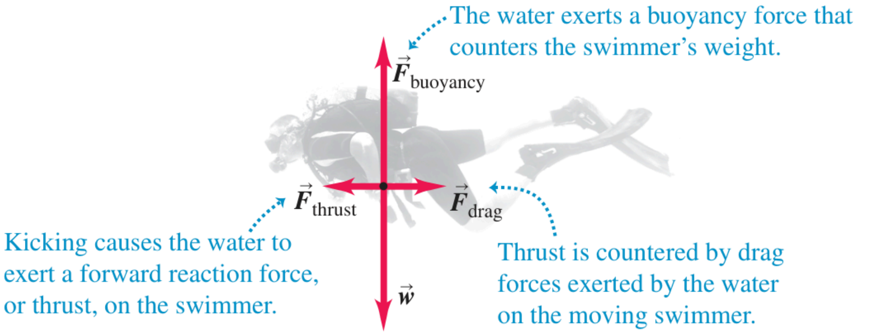


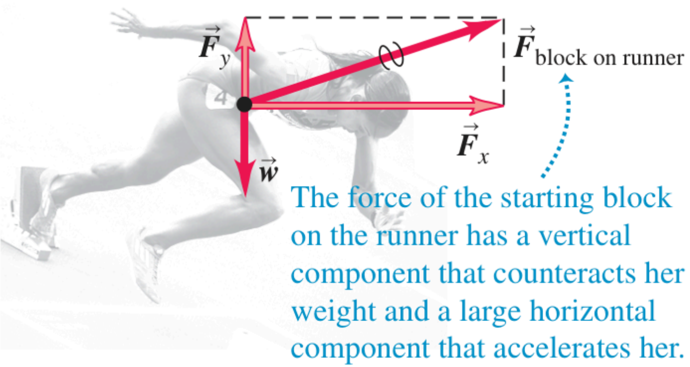
- When a problem involves more than one object, we take the problem apart and draw a separate free-body diagram for each





- Some real-life situations and corresponding free-body diagrams





The force of the starting block on the runner has a vertical component that counteracts her weight and a large horizontal component that accelerates her.

This player is a freely falling object.

To jump up, this player is pushing down against the floor, increasing the upward reaction force $\vec{n}$ of the floor on him.

Inertial frame of reference

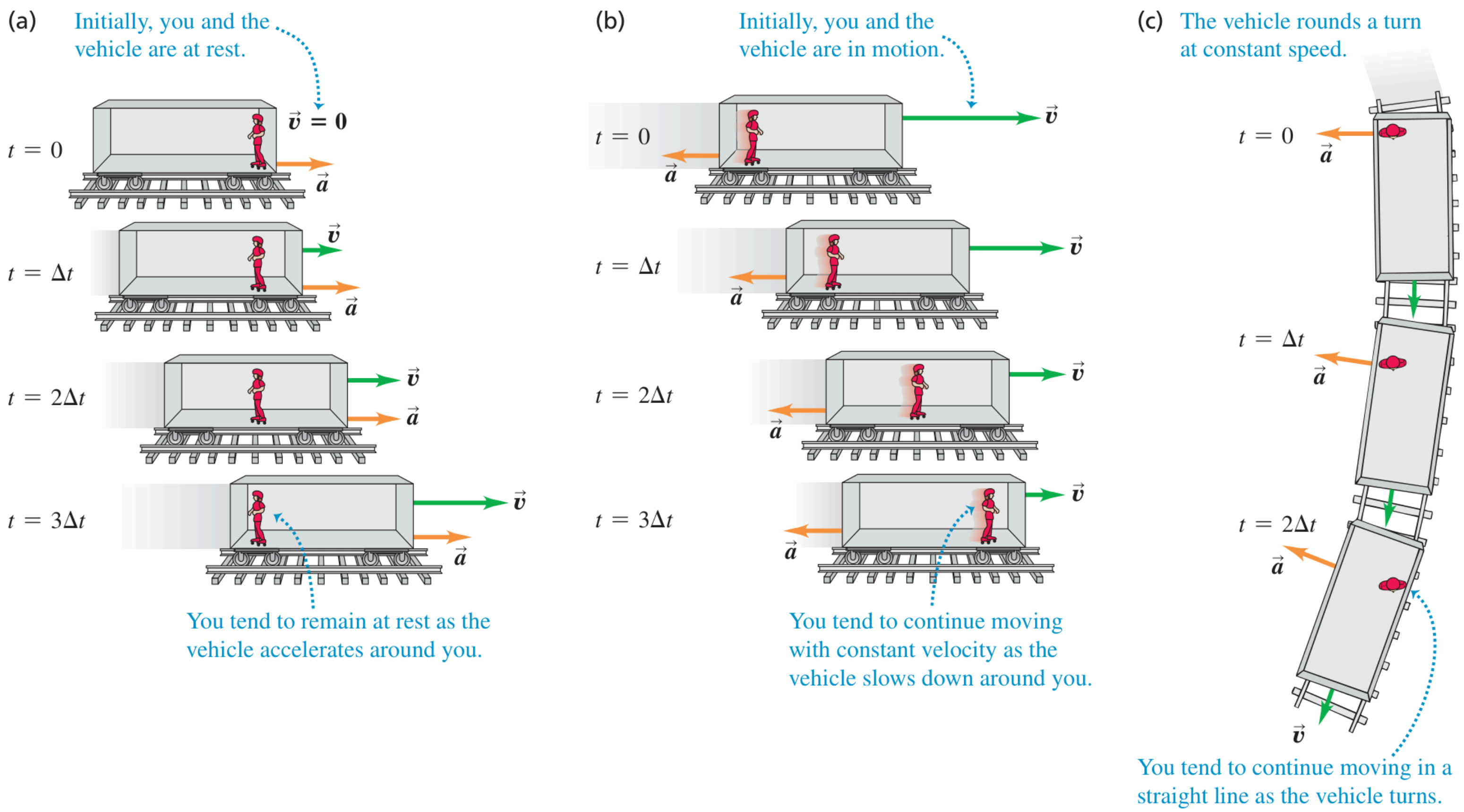
- Recall the concept of **reference frame** in kinematics
  - eg. 🧑 🛼 🚃 $\rightarrow \longrightarrow$ (accelerating)
  - Relative to people in the train, you start moving backwards from rest as train gains speed. Relative to people outside, you remain at rest. Newton's first law applies differently?
  - Relative to train frame, the first law says you don't move with constant velocity $\rightarrow$ you accelerate $\rightarrow$ net external force $\neq 0$
  - Relative to earth frame, the first law says you remain at rest $\rightarrow$ you don't accelerate $\rightarrow$ net external force on you $= 0$

- Here, there's a more valid or suitable frame for Newton's laws

- That'd be **inertial frame of reference**, which is a frame that is not accelerating or rotating relative to another frame
  - Basically, it is where first law holds true. For practicality, a frame fixed to earth can be approximated as an inertial frame, though earth's rotation and orbit mean it isn't strictly inertial

- There is no single inertial frame of reference that is preferred over all others for formulating Newton's laws
  - If one frame is inertial, then every other frame moving relative to it with constant velocity is also inertial

- eg. we pick inertial (earth) over non-inertial (accelerating train):



Notes on units and magnitudes

- In British system, unit of force is pound/pound-force (lb/lbf), unit of mass is slug, unit of distance is foot (ft), thus 1 lb = 1 slug ft/s^2
  - Official definition of pound is 1 lb = 4.448221615260 N

[1]

[1]Despite its name, the British unit of mass has nothing to do with the type of slug shown here. A common garden slug has a mass of about 15 grams, or about 10^{-3} slug.

Notes on units and magnitudes

- What we have been using is SI or metric system (m, kg, s; N)
- In cgs metric system, unit of mass is gram (g), equal to 10^{-3} kg, and unit of distance is centimeter (cm), equal to 10^{-2} m
  - Unit of force is dyne (dyn): $1 \text{ dyn} = 1 \text{ g cm/s}^2 = 10^{-5}$ N
- Handy to know!
  - A pound is about 4.4 N and a newton is about 0.22 pound
  - An object with a mass of 1 kg has a weight of about 2.2 lb at earth's surface

Typical force magnitudes in newtons (N)		
☀️	Sun's gravitational force on the earth	3.5×10^{22}
🐋	Weight of a large blue whale	1.9×10^{6}
🚂	Maximum pulling force of a locomotive	8.9×10^{5}
🏉	Weight of a 250 lb linebacker	1.1×10^{3}
🍎	Weight of a medium apple	1
🪰	Weight of the smallest insect eggs	2×10^{-6}
⚡	Electric attraction bet. p^+ and e^- in H atom	8.2×10^{-8}
🦠	Weight of a very small bacterium	1×10^{-18}
⚛️	Weight of a hydrogen atom	1.6×10^{-26}
🪐	Gravitational attraction bet. p^+, e^- in H atom	3.6×10^{-47}

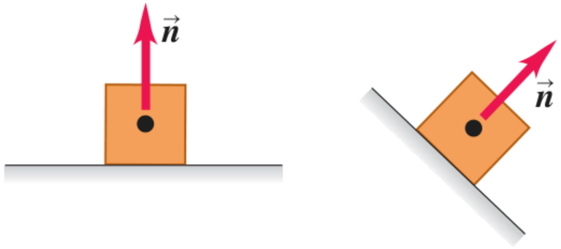
Questions? 😦

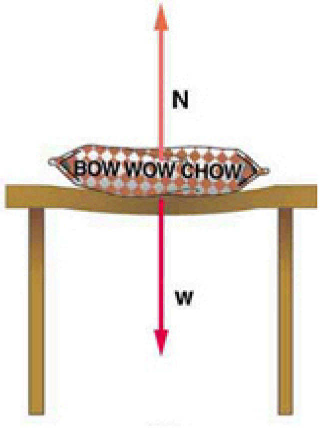
Applying Newton's laws ⚖️

Common types of forces

- **Normal force $\vec{n}$** is exerted on an object by any surface with which it is in contact. Normal means that the force always acts perpendicular to the surface of contact, no matter what angle
  - Generally, $\vec{n} = -\vec{w} = -m\vec{g}$

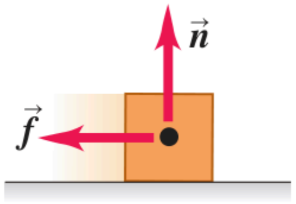
(a) **Normal force $\vec{n}$:** When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.

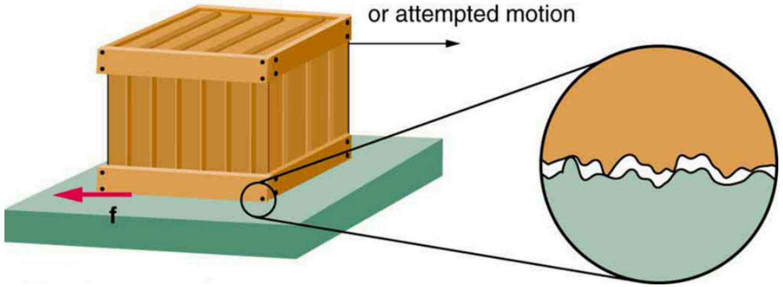




- **Friction force** $\vec{f}$ exerted on an object by a surface acts parallel to the surface, in the direction that opposes sliding
  - When an object is sliding over surface, the friction force is called kinetic friction, otherwise it is static friction

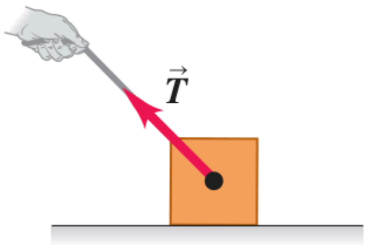
(b) **Friction force $\vec{f}$:** In addition to the normal force, a surface may exert a friction force on an object, directed parallel to the surface.

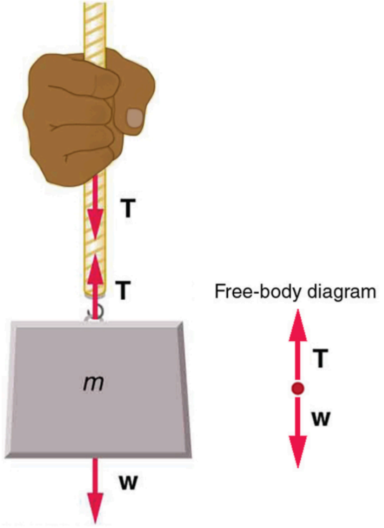


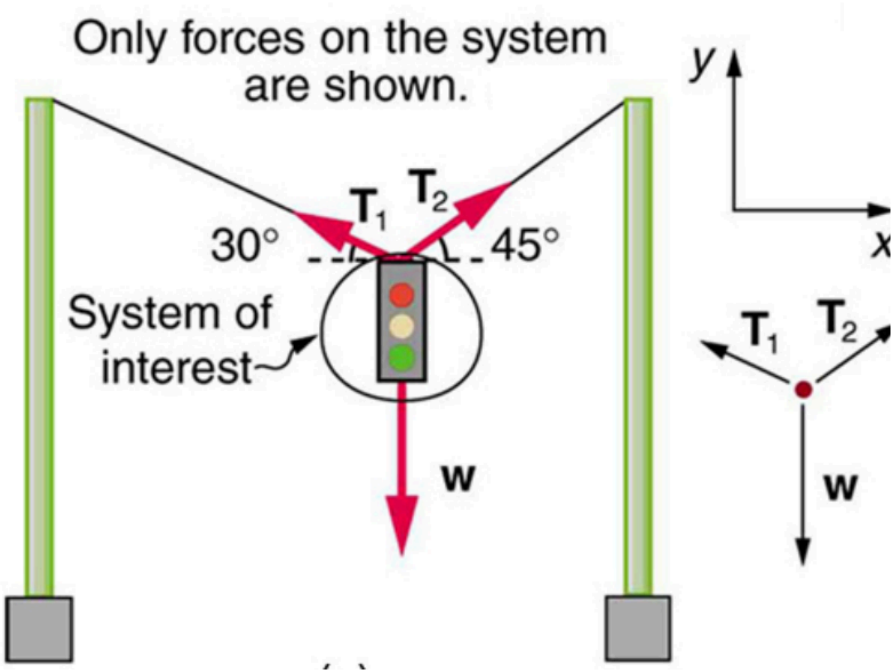


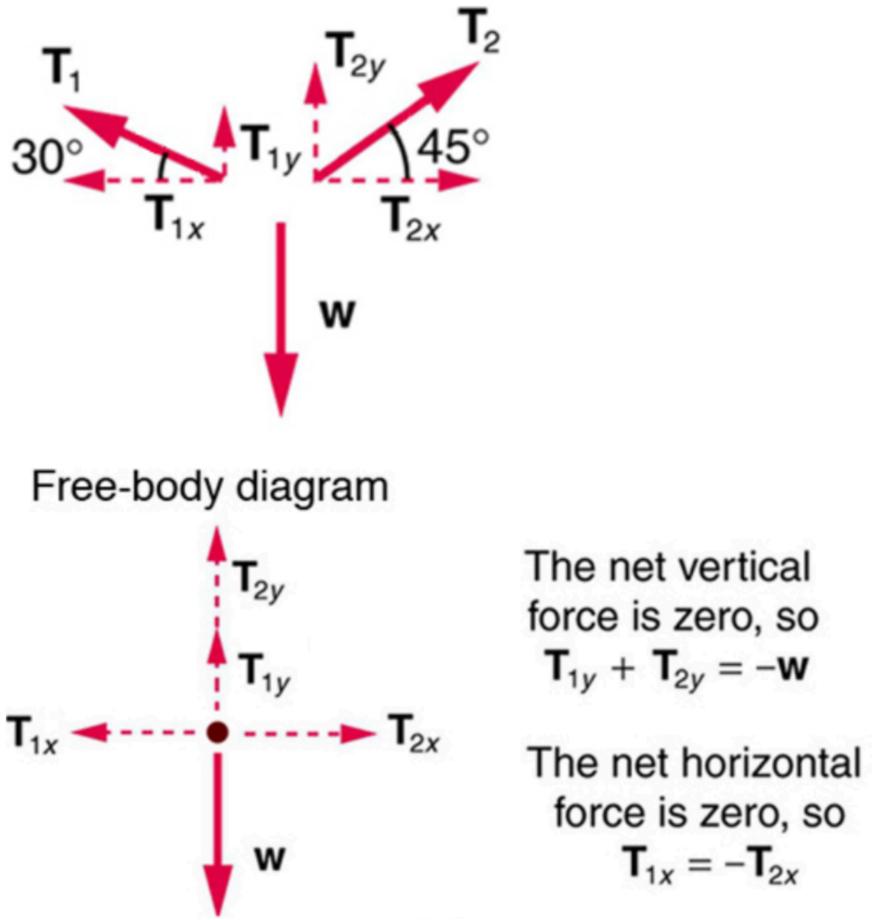
- The pulling force exerted by a stretched rope or cord on an object to which it's attached is **tension force** $\vec{\boldsymbol{T}}$
  - $\vec{T} = -\vec{w}$ if object of interest is hanging straight down via a simple rope

(c) **Tension force $\vec{T}$:** A pulling force exerted on an object by a rope, cord, etc.



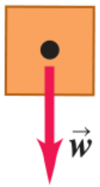


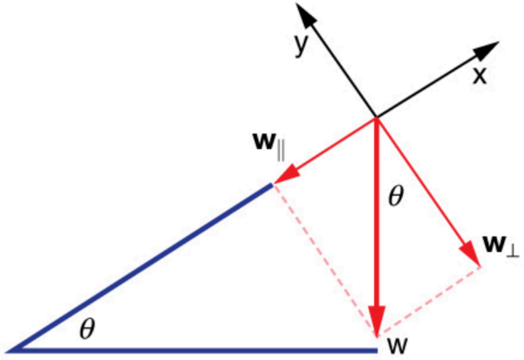

Only forces on the system are shown.
y
x
T1
T2
30°
45°
System of interest
w
T1
T2
w


T1
T2y
T2
30°
T1y
45°
T1x
T2x
w
Free-body diagram
T2y
T1y
T1x
T2x
w
The net vertical force is zero, so
T1y + T2y = −w
The net horizontal force is zero, so
T1x = −T2x

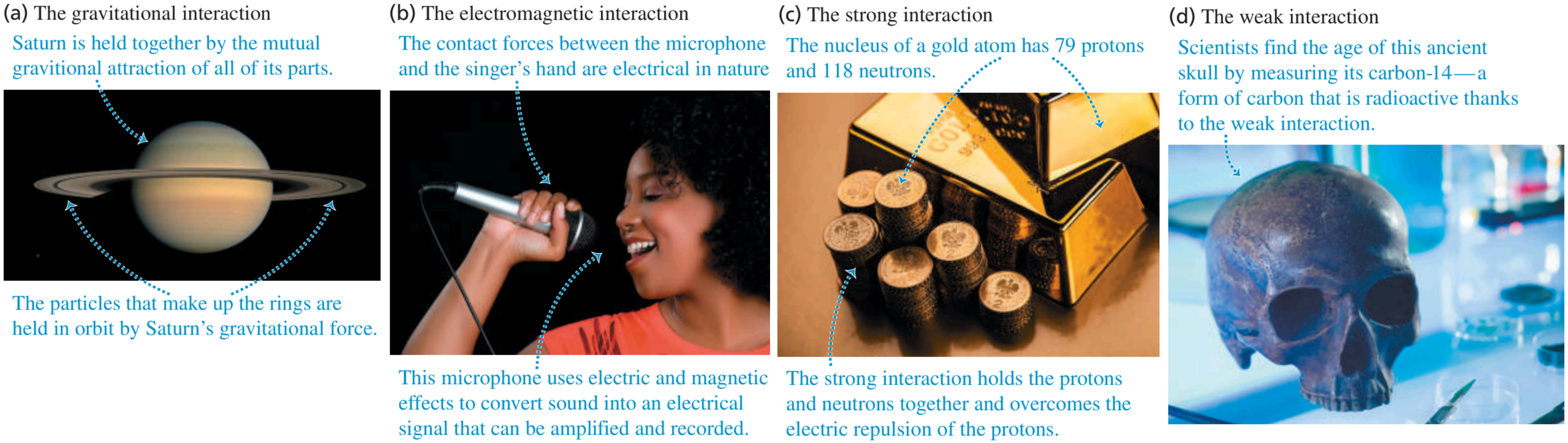
- The earth pulls a dropped object toward it even though there is no direct contact between object and earth. This gravitational force that earth exerts on object is the object's **weight** $\vec{\boldsymbol{w}}$
  - On an incline with angle θ, weight $\vec{w}$ can break into parts that act perpendicular ($\vec{w}_\perp$) and parallel ($\vec{w}_\parallel$) to the surface:
    $\vec{w}_\parallel = w \sin\theta = mg \sin\theta, \qquad \vec{w}_\perp = w\cos\theta = mg\cos\theta$

(d) **Weight $\vec{w}$:** The pull of gravity on an object is a long-range force (a force that acts over a distance).





- Forces that are not really giving "daily life" vibes, but are still common, are the **fundamental forces** of nature: gravitational, electromagnetic, strong, and weak interactions



Questions? 😄

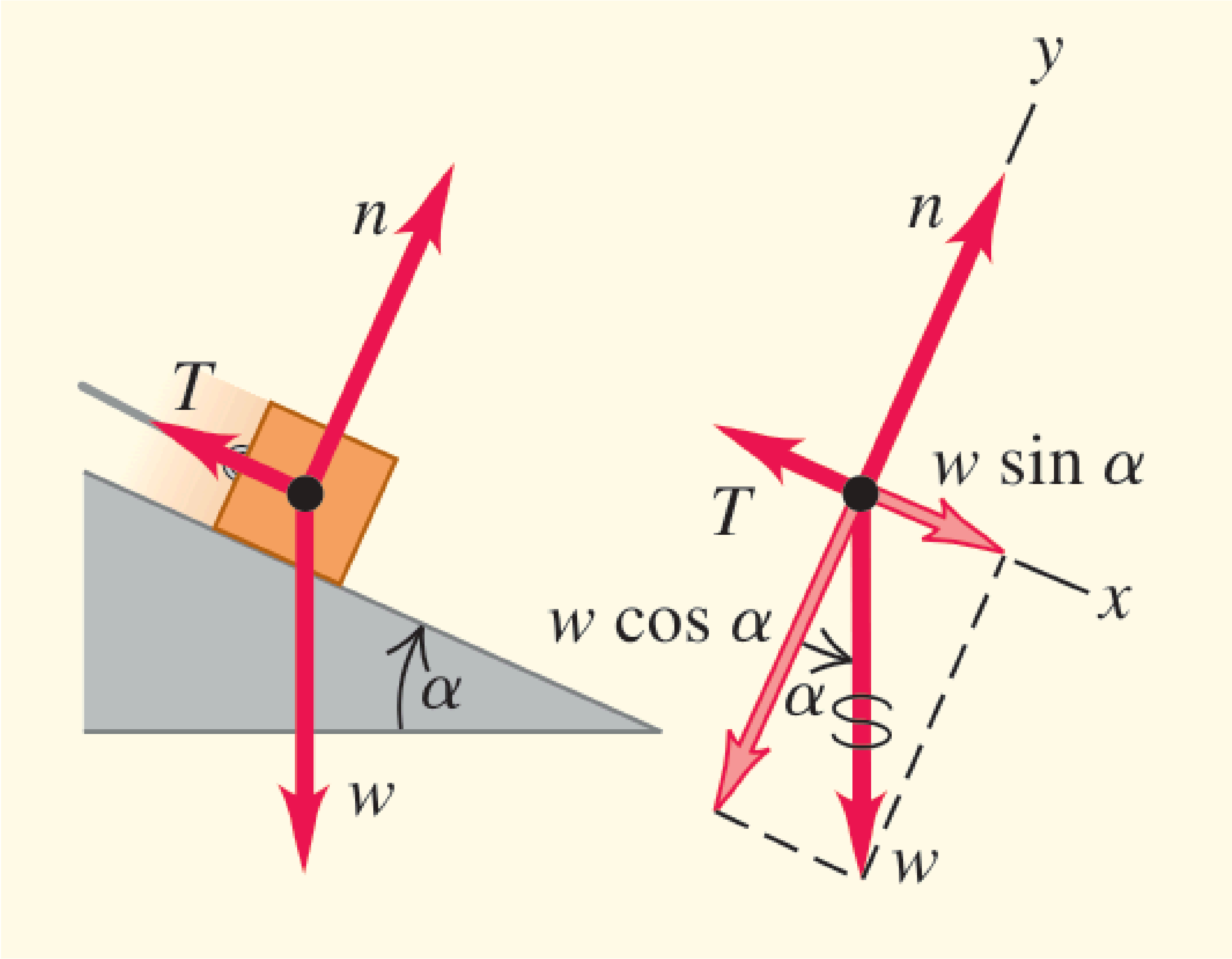
Quick tips

- Newton's first and second laws apply to a specific object
  - Decide which object you are referring when using these laws
- Only forces acting on the object matter
  - $\sum \vec{F}$ includes all forces that act on the object in question
  - Don't confuse forces acting on an object with the forces exerted by that object on some other object
- Free-body diagrams are essential to help identify relevant forces
  - Show the force vectors that act on the chosen object "free" of its surroundings

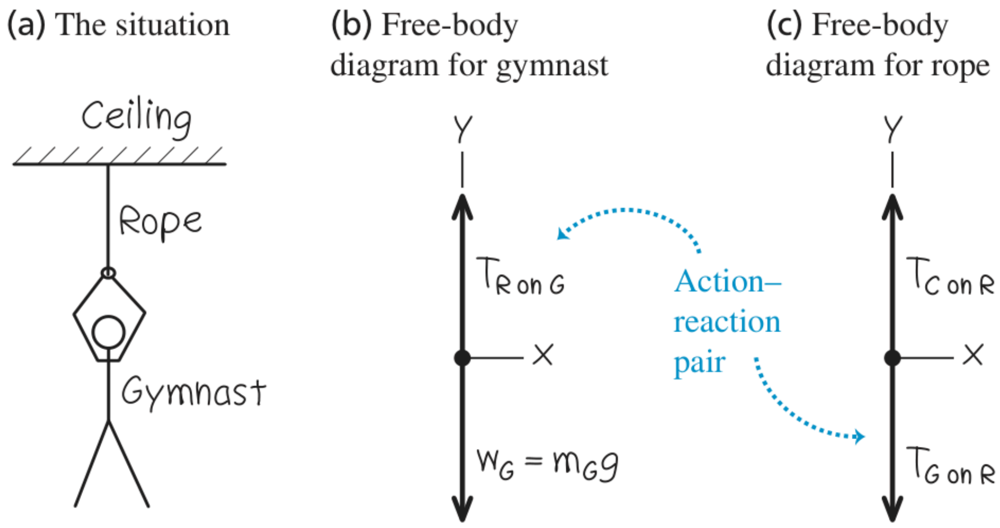
Using the first law: objects in equilibrium 🅰️

$$\sum \vec{F} = \vec{0}, \qquad \sum F_x = 0, \quad \sum F_y = 0$$

- When an object is in equilibrium in an inertial frame of reference, that is either at rest or moving with constant velocity, the vector sum of forces acting on it must be zero (first law)
  - eg. hanging lamp, kitchen table, ✈️ ➡️ at constant $\vec{v}$
- Third law is also frequently needed in equilibrium problems
  - The 2 forces in action-reaction pair never act on same object


n
T
α
w
y
n
w sin α
T
x
w cos α
α
w

Example. A gymnast with mass $m_G = 50.0$ kg suspends herself from the lower end of a hanging rope of negligible mass. The upper end of the rope is attached to the gymnasium ceiling. (a) What is the gymnast's weight? (b) What force (magnitude and direction) does rope exert on her? (c) What is the tension at the top of rope?



- (a) The magnitude of gymnast's weight is her mass times acceleration due to gravity:

$$w_G = m_G g = (50.0\ \text{kg})(9.80\ \text{m/s}^2) = 490\ \text{N}$$

- (b) The gravitational force on gymnast (her weight) points in $-y$-direction so its y-component is $-w_G$. The upward force of rope on gymnast has unknown magnitude $T_{R \text{ on } G}$ and $+y$-component. Using first law,

$$\text{gymnast:} \qquad \sum F_y = T_{R \text{ on } G} + (-w_G) = 0 \qquad \text{so}$$
$$T_{R \text{ on } G} = w_G = 490 \text{ N}$$

  - The rope pulls upward on gymnast with force $T_{R \text{ on } G}$ of magnitude 490 N

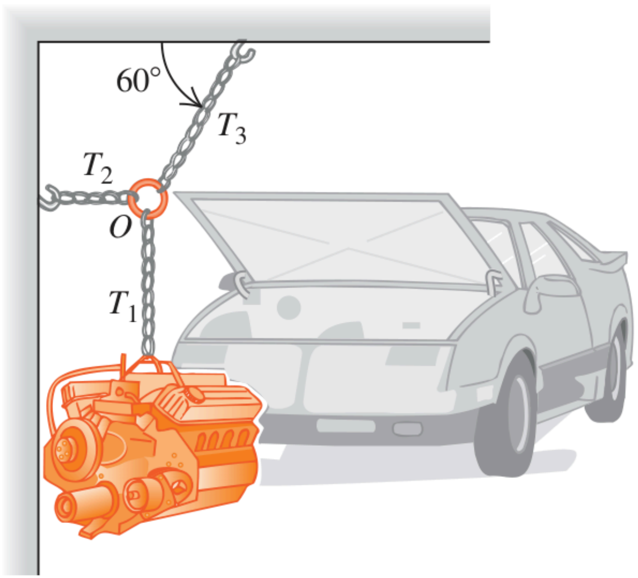
- (c) Assume rope is weightless, so only forces on it are those exerted by ceiling (upward tension $T_{C \text{ on } R}$) and by gymnast (downward $T_{G \text{ on } R} = 490$ N). From first law, net vertical force on rope in equilibrium must be zero, so

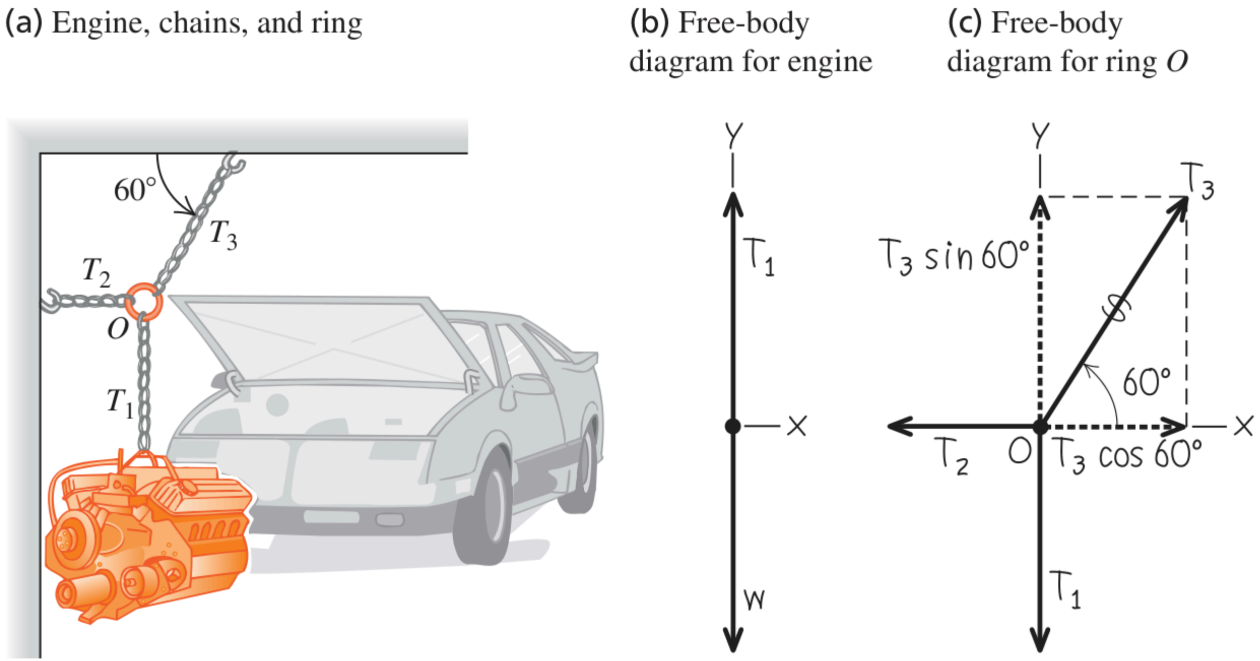
$$\text{rope:} \qquad \sum F_y = T_{C \text{ on } R} + (-T_{G \text{ on } R}) = 0 \qquad \text{so}$$

$$T_{C \text{ on } R} = T_{G \text{ on } R} = 490 \text{ N}$$

- Key concept: The sum of all the external forces on an object in equilibrium is zero. The tension has the same value at either end of a rope or string of negligible mass

Example. In figure, a car engine with weight w hangs from a chain that is linked at ring O to two other chains, one fastened to the ceiling and the other to the wall. Find expressions for the tension in each of three chains in terms of w. The weights of ring and chains are negligible compared with the weight of the engine.





- Forces acting on engine are along y-axis only, so first law says

$$\text{engine:} \qquad \sum F_y = T_1 + (-w) = 0 \qquad \text{and} \qquad T_1 = w$$

- Horizontal and slanted chains don't exert forces on engine itself as they are not attached to it. In ring fbd, recall that T_1, T_2 and T_3 are magnitudes of forces. Resolving them into their x- and y-components gives us two equations for ring:

$$\text{ring:} \qquad \sum F_x = T_3 \cos 60° + (-T_2) = 0$$

$$\text{ring:} \qquad \sum F_y = T_3 \sin 60° + (-T_1) = 0$$

- Since $T_1 = w$ from engine eq, we can rewrite second ring eq:

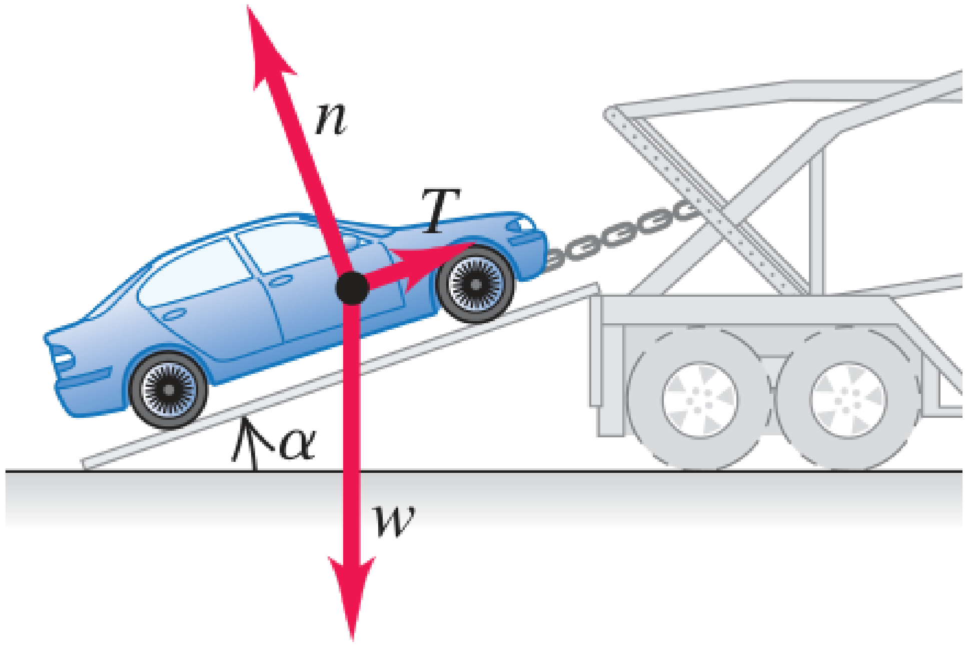
$$T_3 = \frac{T_1}{\sin 60°} = \frac{w}{\sin 60°} = 1.2w$$

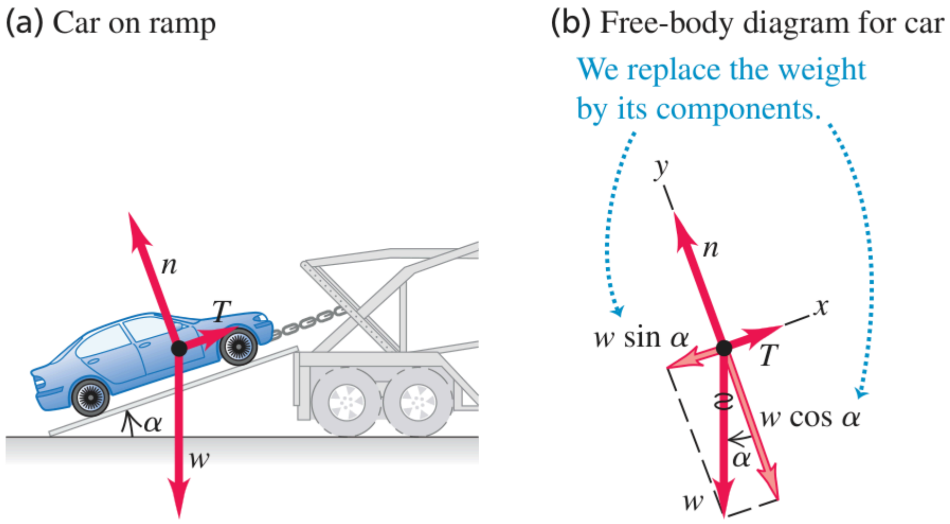
- Using this result in first ring equation:

$$T_2 = T_3 \cos 60° = w \frac{\cos 60°}{\sin 60°} = 0.58w$$

- Key concept: In two-dim problems, always write two force equations for each object: one for the x-components of the forces and one for the y-components of the forces

Example. A car of weight w rests on a slanted ramp attached to a trailer. Only a cable running from the trailer to the car prevents the car from rolling off the ramp. The car's brakes are off and its transmission is in neutral. Find the tension in the cable and the force that the ramp exerts on the cars tires.





- We must first find the x- and y-components of weight $\vec{w}$. We consider the right triangles and get corresponding $w_{\parallel}$ along x and $w_{\perp}$ along y. Both components are negative so,

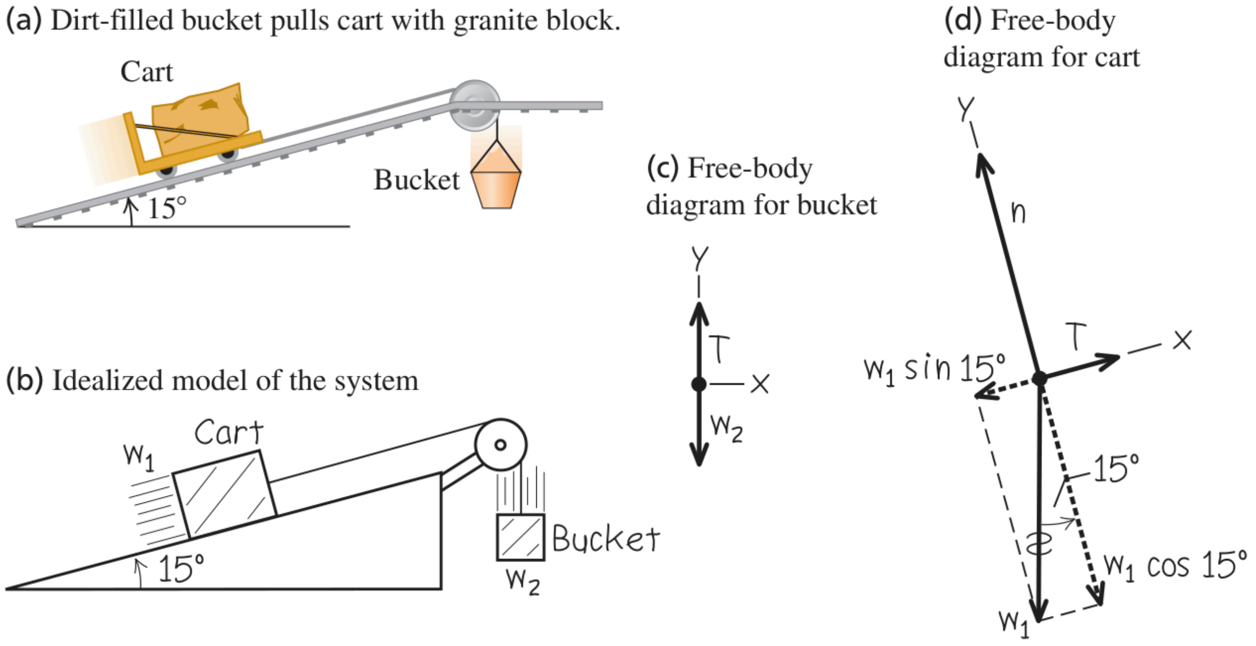
$$w_x = w_{\parallel} = -w \sin \alpha, \qquad w_y = w_{\perp} = -w \cos \alpha$$

- Newton's first law then gives us

$$\sum F_x = T + (-w \sin\alpha) = 0,$$
$$\sum F_y = n + (-w \cos\alpha) = 0$$
$$\implies T = w \sin\alpha, \qquad n = w \cos\alpha$$

- Quick check: If the ramp is horizontal ($\alpha = 0°$), we get $T = 0$ and $n = w$, which for the former means no cable tension T is needed to hold the car. If the ramp is vertical ($\alpha = 90°$), we get $T = w$ and $n = 0$, which means the cable tension T supports all of car's weight, and there's nothing pushing car against ramp

Example. Your firm needs to haul granite blocks up a 15° slope out of a quarry and to lower dirt into the quarry to fill the holes. You design a system in which a granite block on a cart with steel wheels (weight w_1 including both block and cart) is pulled uphill on steel rails by a dirt-filled bucket (weight w_2 including both dirt and bucket) that descends vertically into the quarry as in figure. How must weights w_1 and w_2 be related in order for the system to move with constant speed? Ignore friction in the pulley and wheels, and ignore the weight of the cable.



- The cart and bucket each move with a constant velocity (in a straight line at constant speed), hence each object is in equilibrium and we can apply the first law to each

- Applying first law to the ..., we find

$$\begin{aligned} \text{bucket}: &\quad \sum F_y = T + (-w_2) = 0 &&\implies T = w_2 \\ \text{cart+block}: &\quad \sum F_x = T + (-w_1 \sin 15°) = 0 &&\implies T = w_1 \sin 15° \end{aligned}$$

- Equating the two equations for T, we find

$$w_2 = w_1 \sin 15° = 0.26 w_1$$

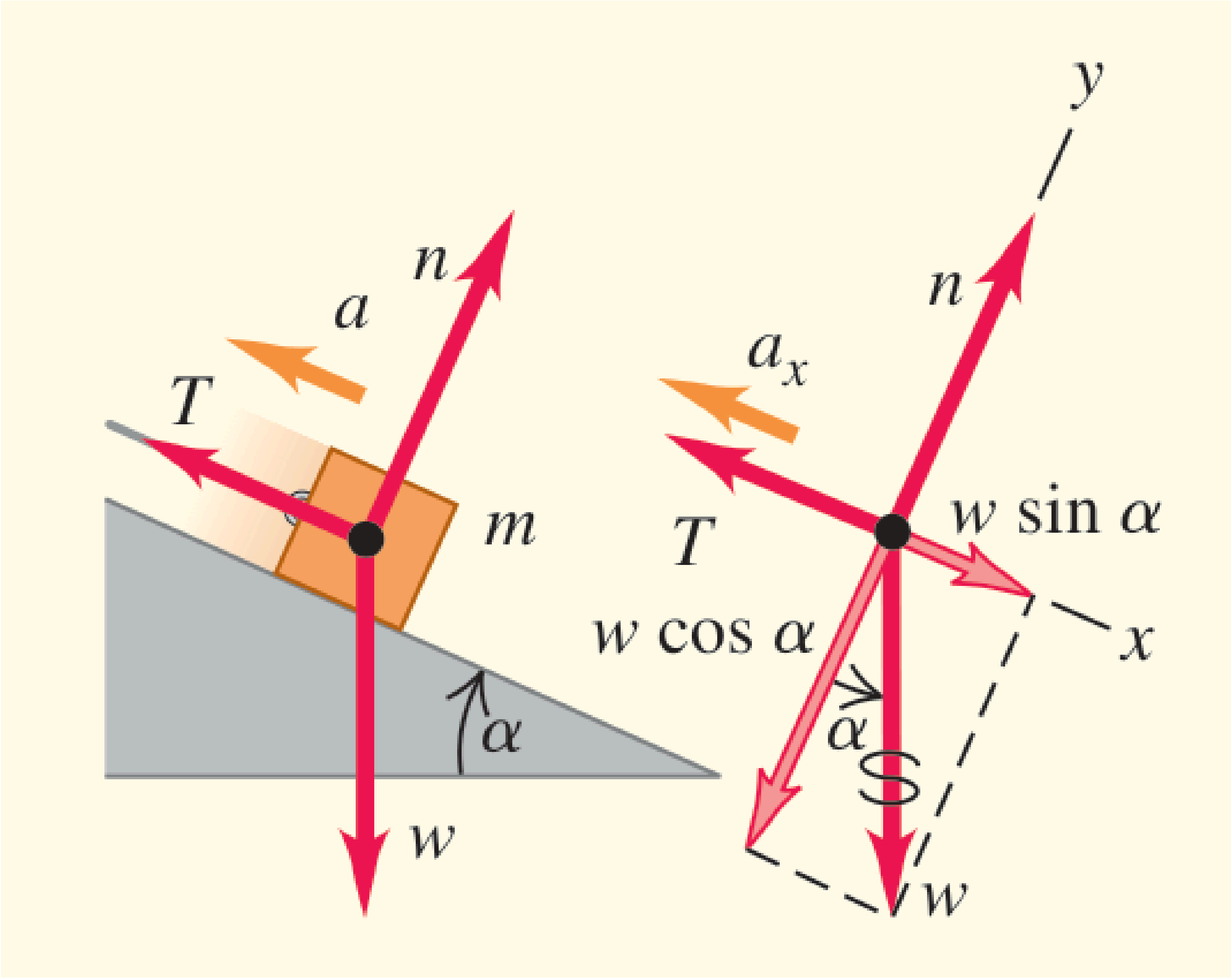
- Bonus observation: The system can move with constant speed in either direction if the weight of the dirt and the bucket is 26% of that of the granite block and cart

Questions? 😏

Using the second law: dynamics of objects 🅱️

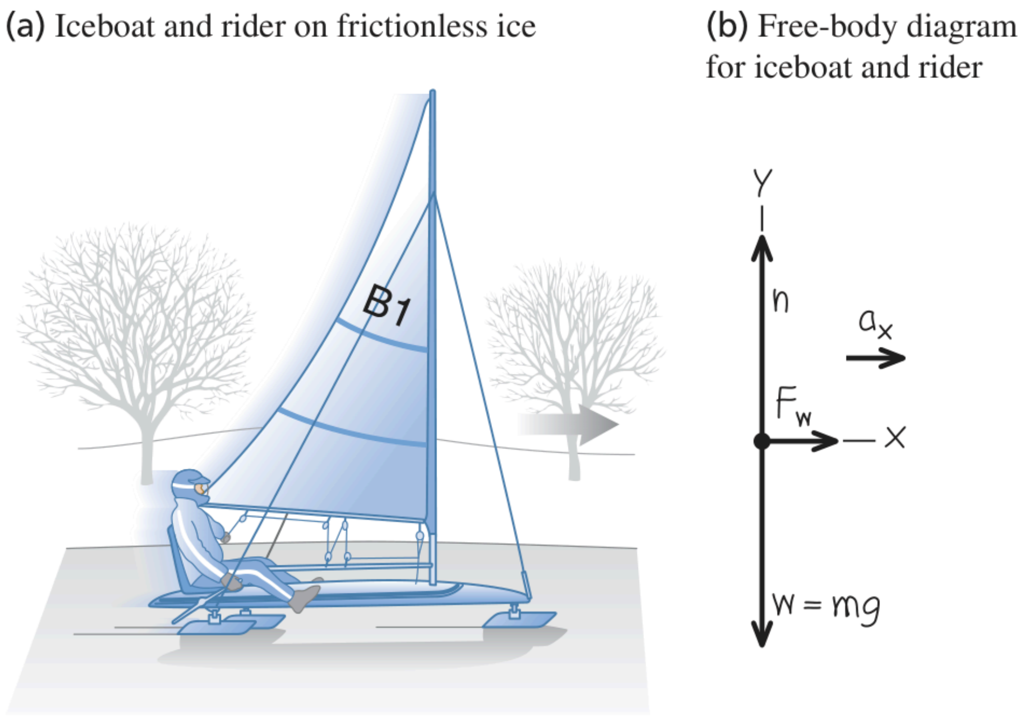
$$\sum \vec{F} = m\vec{a}, \qquad \sum F_x = ma_x, \quad \sum F_y = ma_y$$

- If the vector sum of forces on an object is not zero, the object accelerates
  - Its acceleration is related to net force by second law
- Just as for equilibrium problems, free-body diagrams are useful, and the normal force exerted on an object is not always equal to its weight



n
a
T
m
α
w
y
n
a_x
T
w sin α
w cos α
x
α
w

Example. An iceboat is at rest on a frictionless horizontal surface as in figure. Due to the blowing wind, 4.0 s after the iceboat is released, it is moving to the right at 6.0 m/s (about 22 km/h). What constant horizontal force F_W does the wind exert on the iceboat? The combined mass of iceboat and rider is 200 kg.



- Force F_W is in $+x$-direction while n and $w = mg$ are in $+y$- and $-y$-directions, respectively. Hence we have

$$\sum F_x = F_W = ma_x,$$

$$\sum F_y = n + (-mg) = 0 \qquad \text{so} \qquad n = wg$$

- To find F_W, we first solve for a_x using the kinematic equation

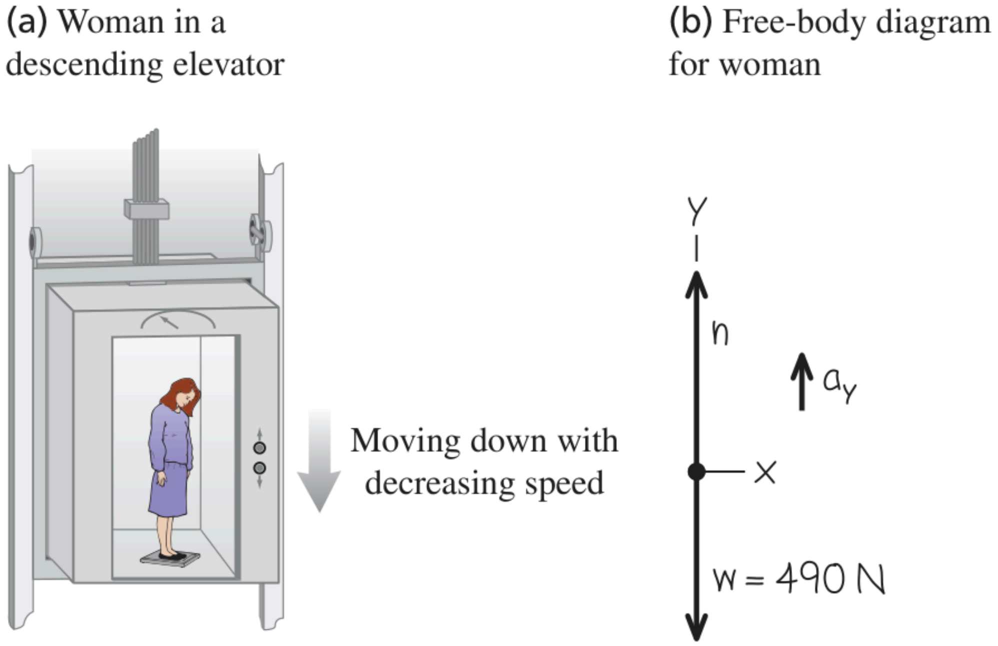
$$v_x = v_{0x} + a_x t$$

$$\implies a_x = \frac{v_x - v_{0x}}{t} = \frac{6.0\ \text{m/s} - 0}{4.0\ \text{s}} = 1.5\ \text{m/s}^2$$

$$\implies F_W = ma_x = (200\ \text{kg})(1.5\ \text{m/s}^2) = 300\ \text{kg m/s}^2 = 300\ \text{N}$$

- Key concept: It's usually best to choose one positive axis to be in the direction of the acceleration

Example. A 50.0 kg woman stands on a bathroom scale while riding in an elevator. The elevator is initially moving downward at 10.0 m/s. It slows to a stop with constant acceleration in a distance of 25.0 m. While the elevator is moving downward with decreasing speed, what is the reading on the scale?



- We start by finding a_y using the kinematic equation

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\implies a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(0)^2 - (-10.0\ \text{m/s})^2}{2(-25.0\ \text{m})} = +2.00\ \text{m/s}^2$$

- The acceleration is upward (positive), just as it should be

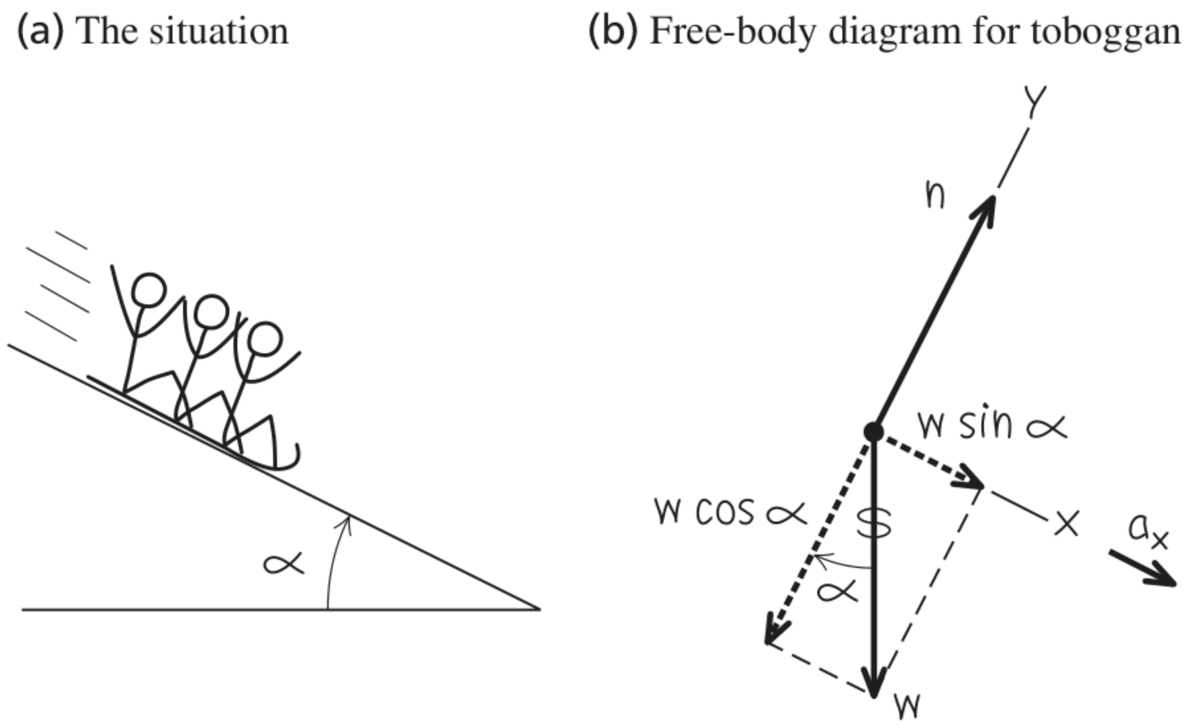
- Second law then gives

$$\sum F_y = n + (-mg) = ma_y \qquad \implies n = mg + ma_y = m(g + a_y)$$

$$n = (50.0\ \text{kg})(9.80\ \text{m/s}^2 + 2.00\ \text{m/s}^2) \qquad = 590\ \text{N}$$

- Key concept: When riding an accelerating vehicle such as an elevator, your **apparent weight** (the normal force that vehicle exerts on you) is in general not equal to your actual weight

Example. A toboggan loaded with students (total weight w) slides down a snow-covered hill that slopes at a constant angle α. The toboggan is well waxed, so there is virtually no friction. What is its acceleration?



- Normal force has only a y-component, but weight has both x- and y-components $w_x = w_\parallel = w \sin\alpha$ and $w_y = w_\perp = -w\cos\alpha$

- Second law in component form then tells us that

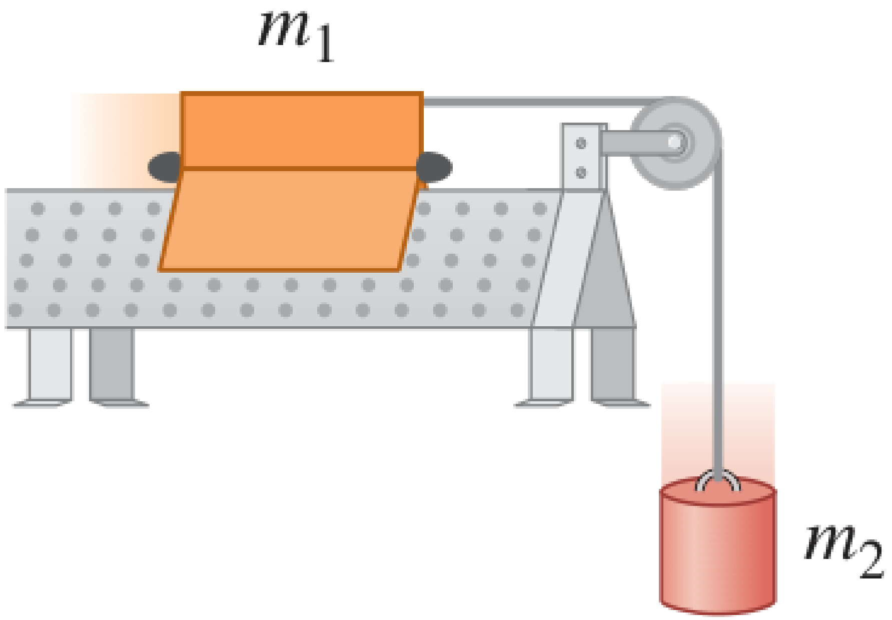
$$\sum F_x = w \sin\alpha = ma_x, \qquad \sum F_y = n - w\cos\alpha = ma_y$$

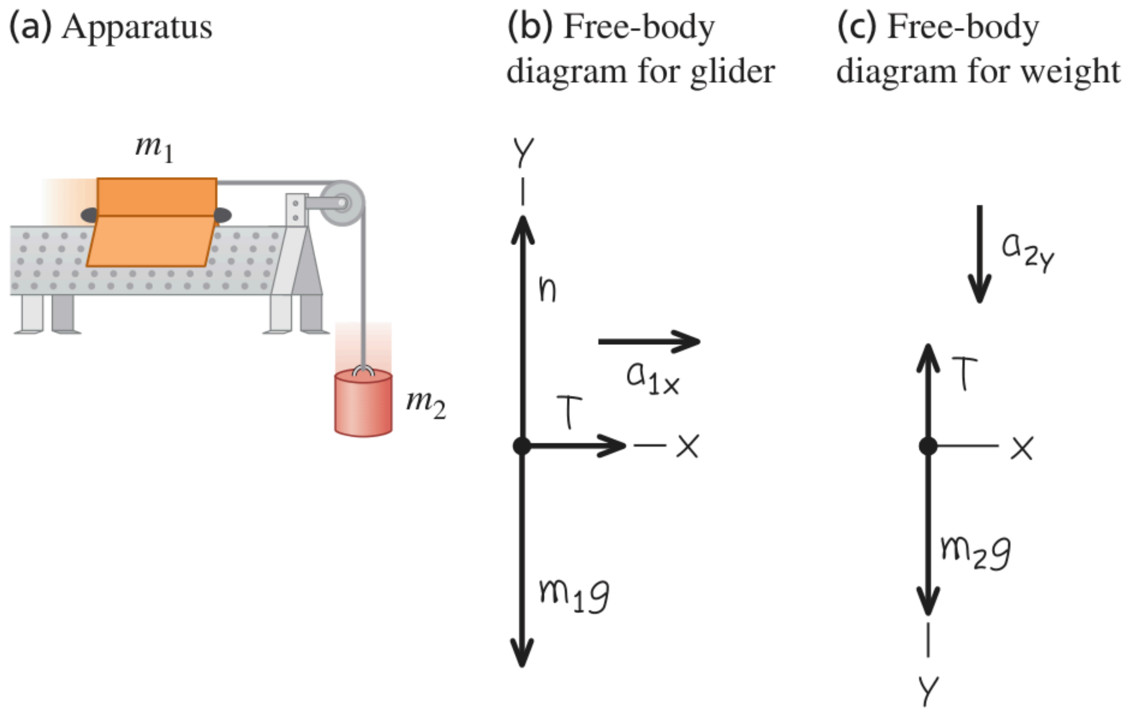
- Since $w = mg$, the x-component equation gives

$$mg \sin\alpha = ma \qquad \Longrightarrow a_x = g\sin\alpha$$

- Note: we didn't need the y-component equation to find accel. a thanks to our choice to make x-axis lie along direction of a
  - The y-equation tells us the magnitude of the normal force exerted by the hill on the toboggan: $n = w\cos\alpha = mg\cos\alpha$

Example. An air-track glider with mass m_1 is moving on a level, frictionless air track in physics lab. The glider is connected to a lab weight with mass m_2 by a light, flexible, nonstretching string that passes over a stationary, frictionless pulley. Find acceleration of each object and tension in the string.





- While the directions of the two accelerations are different, their magnitudes are the same

  - This is because the string doesn't stretch so the two objects must move equal distances in equal times and their speeds at any instant must be equal. Thus, $a_{1x} = a_{2y} = a$

- Second law gives

$$\begin{aligned} \text{glider}: &\quad \sum F_x = T = m_1 a_{1x} = m_1 a \\ \text{glider}: &\quad \sum F_y = n + (-m_1 g) = m_1 a_{1y} = 0 \\ \text{lab weight}: &\quad \sum F_y = m_2 g + (-T) = m_2 a_{2y} = m_2 a \end{aligned}$$

- The x-equation for glider and lab weight give us

$$\text{glider}: \qquad T = m_1 a$$
$$\text{lab weight}: \qquad m_2 g + (-T) = m_2 a$$

- We add these two to elimiate T giving

$$m_2 g = m_1 a + m_2 a = (m_1 + m_2)a$$
$$\implies a = \frac{m_2}{m_1 + m_2} g$$

which subbing back into $T = m_1 a$ yield

$$T = \frac{m_1 m_2}{m_1 + m_2} g$$

Questions? 🙄

Quiz time 🕐

Jaw injury

Due to a jaw injury, a patient must wear a strap, as in the figure, that produces a net upward force of 5.00 N on his chin. The tension is the same throughout the strap. To what tension must the strap be adjusted to provide the necessary upward force?